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Ship Geometry

Section 1
Ships' Lines

1.1 Delineation and Arrangement of Lines Drawing. The exterior form of a ship's hull is a curved surface defined by the lines drawing, or simply "the lines." Precise and unambiguous means are needed to describe this surface, inasmuch as the ship's form must be configured to accommodate all internals, must meet constraints of buoyancy, stability, speed and power, and seakeeping, and must be "buildable." Hence, the lines consist of orthographic projections of the intersections of the hull form with three mutually perpendicular sets of planes, drawn to a suitable scale.

Fig. 1 shows a lines drawing for a single-screw cargo-passenger ship.

The profile or *sheer plan* shows the hull form intersected by the centerplane—a vertical plane on the ship's centerline—and by buttock planes which are parallel to it, spaced for convenient definition of the vessel's shape and identified by their distance off the centerplane. The centerplane intersection shows the profile of the bow and stern. Below the profile is the *half-breadth* or *waterlines plan*, which shows the intersection of the hull form with planes parallel to the horizontal baseplane, which is called the base line. All such parallel planes are called waterline planes, or waterplanes. It is convenient to space most waterplanes equally by an integral number of meters (or feet and inches), but a closer spacing is often used near the baseline where the shape of hull form changes rapidly. DWL represents the design waterline, near which the fully loaded ship is intended to float. All waterlines are identified by their height above the baseline.

The *body plan* shows the shapes of sections determined by the intersection of the hull form with planes perpendicular to the buttock and waterline planes. In Fig. 1 this is shown above the profile, but it might otherwise be drawn to the right or left of the profile, using a single extended molded baseline, depending upon the width and length of paper being used. Alternatively, the body plan is sometimes superimposed on the profile, with the body plan's centerplane midway between the ends of the ship in profile view. Planes defining the body plan are known as body plan stations. They are usually spaced equally apart, such that there are 10 spaces—or multiples thereof—in the length of the ship, but with a few extra stations at the ends of the ship at one half or one quarter this spacing.

Most ships are symmetrical about the centerplane, and the lines drawing shows waterlines in the half-breadth plan on only one side of the centerline. Asymmetrical features on some ships, such as overhanging flight decks on aircraft carriers, must be depicted separately. Correspondingly, the body plan shows sections on one side of the centerline only—those in the forebody on the right hand side and those in the afterbody on the left. By convention in the U.S., the bow of the ship is shown to the right. With the arrangement of the lines as shown in Fig. 1, the drawing represents a case of first angle projection in descriptive geometry.

The lines in Fig. 1 represent the *molded surface* of the ship, a surface formed by the outer edges of the frames, or inside of the "skin," in the case of steel, aluminum and wooden vessels. In the case of glass reinforced plastic vessels, the molded surface is the outside of the hull. (The term molded surface undoubtedly arose from the use of wooden "molds" set up to establish a surface in space to which frames could be formed when wooden vessels were being built).

The shell plating of a steel or aluminum ship constitutes the outer covering of the molded surface. The shell plating is relatively thin and is formed of plates that are usually of varying thickness, causing some unevenness, although the molded surface is generally smooth and continuous.

The thickness of planking of a wooden boat is relatively larger than the shell thickness of a steel vessel, and it is the usual practice to draw the lines of a wooden boat to represent the surface formed by the outside of the planking, since this gives the true external form. However, for construction purposes it is necessary to deal with the molded form, and therefore it is not unusual to find the molded form of wooden vessels delineated on a separate lines drawing.

In the sheer plan of Fig. 1, the base line, representing the bottom of the vessel, is parallel to the DWL, showing that the vessel is designed for an "even-keel" condition. Some vessels—especially tugs and fishing vessels—are often designed with the molded keel line raked downward aft, giving more draft at the stern than the bow when floating at the DWL; such vessels are said to have a designed *drag to the keel.*

1.2 Perpendiculars; Length Between Perpendiculars. A vertical line in the sheer plan of Fig. 1 is drawn at the intersection of the DWL, which is often the estimated summer load line (defined subsequently), and the forward side of the stem. This is known as the *forward perpendicular*, abbreviated as FP. A slight inconsistency is introduced by this definition of FP in that the forward side of the stem is generally in a surface exterior to the molded form by the thickness of contiguous shell plating—or by the stem thickness itself if the stem is of rolled plate.

A corresponding vertical line is drawn at the stern, designated the *after perpendicular* or AP. When there is a rudder post the AP is located where the after side of the rudder post intersects the DWL. In Fig. 1 the AP is drawn at the centerline of the rudder stock, which is the customary location for merchant ships without a well defined sternpost or rudder post. In the case of naval ships, it is customary to define the AP at the after end of the vessel on the DWL. Such a location is also sometimes chosen for merchant vessels—especially vessels with a submerged stern profile extending well abaft the rudder. Fig. 2 shows the various locations of the AP here described.

An important characteristic of a ship is its length between perpendiculars, sometimes abbreviated LBP or *Lpp*. This represents the fore-and-aft distance between the FP and AP, and is generally the same as the length L defined in the American Bureau of Shipping *Rules for Building and Classing Steel Vessels* (Annual)[1]. However, in the *Rules* there is included the proviso that L, for use in the Rules, is not to be less than 96 percent and need not be greater than 97 percent of the length on the summer load line. The summer load line is the deepest waterline to which a merchant vessel may legally be loaded during the summer months in certain specified geographical zones. Methods for determining the summer load line are covered in the discussion of freeboard in *Ship Design and Construction* (Taggart, 1980).

When comparing different designs, a consistent method of measuring ship lengths should be used. Overall length is invariably available from the vessel's plans and LBP is usually also recorded. However, for hydrodynamic purposes, length on the prevailing waterline may be significant; alternatively, an "effective length" of the underwater body for resistance considerations is sometimes required.

One useful method of determining the after end of effective length is to make use of a sectional area curve, whose ordinates represent the underwater cross sectional area of the vessel up to the DWL at a series of stations along its length. (See Section 1.7.) The effective length is usually considered as the overall length of the sectional area curve. However, if the curve has a concave ending, a straight line from the midship-cross-sectional area can be drawn tangent to the curve, as shown in Fig. 3. The intersection of this straight line tangent with the baseline of the graph may then be considered to represent the after end of the effective length. On many single-screw designs it has been found that the point so determined is close to the location of the AP. Such an effective length ending might then be used in calculating hull form coefficients, as discussed in Section 3. A similar definition for the forward end of effective length might be adopted for ships with protruding bulbous bows extending forward of the FP.

It is important that in all calculations and measurements relating to length, the method of determining the length used, and the location of its extremities be clearly defined.

1.3 Midship Section; Parallel Middle Body. An important matter for any ship is the location and shape of the midship cross section, generally designated by the symbol ⊗, which was originally used to indicate the fullest cross section of the vessel. In some of the early sailing ships this fullest section was forward of the midlength, and in some high-speed ships and sailing yachts, the fullest section under water is somewhat abaft the midlength. In any case, the usual practice in modern commercial vessels of most types is to locate ⊗ halfway between the perpendiculars, while in naval ships it is usually midway between the ends of the DWL.

In many modern vessels, particularly cargo vessels, the form of cross section below the DWL amidships extends without change for some distance forward and aft, usually including the midship location. Such vessels are said to have parallel middle body. The ship in Fig. 1 has no parallel middle body, but the form of section under water changes but slightly for small distances forward or abaft the fullest section, which is located amidships.

1.4 Body Plan Stations; Frame Lines; Deck Lines. In order to simplify the calculation of underwater form characteristics, it is customary to divide the LBP into 10—or 20, or 40—intervals by the body plan planes. The locations of these planes are known as body plan stations, or simply stations, and are indicated by straight lines drawn in the profile and half-breadth plans at right angles to the vessel's baseline and centerline, respectively. The intersections of these planes

[1] Complete references are listed at end of chapter.

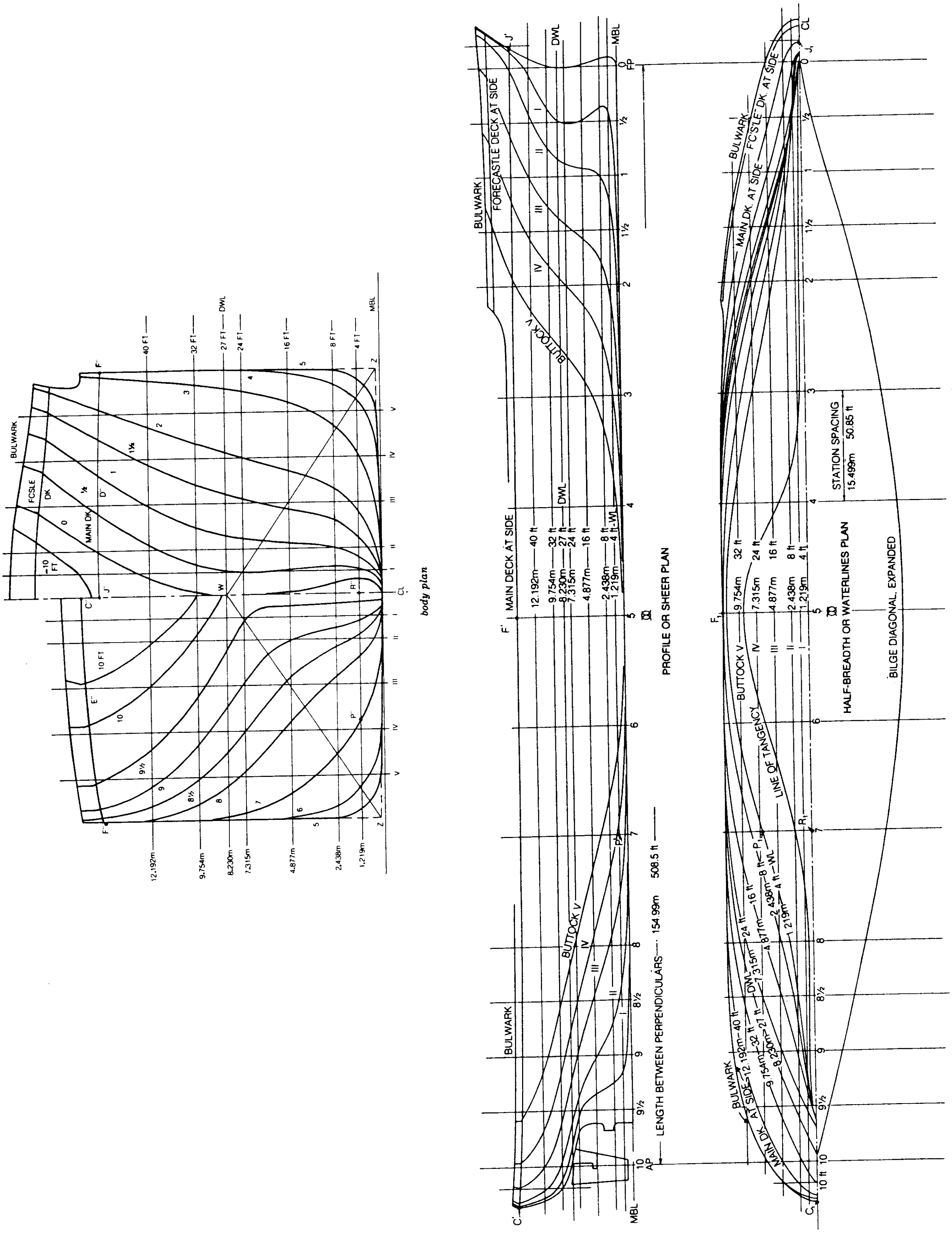


Fig. 1 Lines drawing

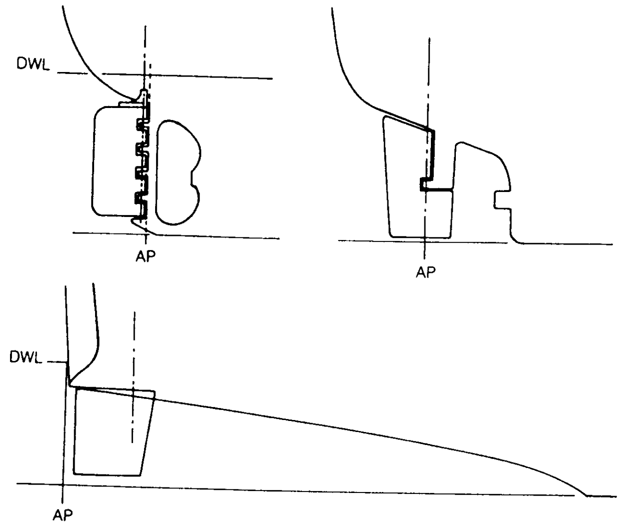


Fig. 2 Alternative locations of after perpendicular

with the molded form appear in their true shape in the body plan.

Body plan stations are customarily numbered from the bow, with the FP designated as station 0. In Europe and Japan, however, station 0 is often located at the AP, with station numbering from aft forward. For the ship shown in Fig. 1, station No. 10 represents the stern extremity of the vessel for calculations relating to the underwater body. It will be noted that additional stations are drawn midway between stations 0 and 1, and 9 and 10, and sometimes between 1 and 2, and 8 and 9, as well. This is done to better define the vessel's form near the ends where it may change rapidly for small longitudinal distances.

Additional stations are often also shown forward of the FP and abaft the AP. These may receive letter or distance designations from the perpendiculars, or a continuation of the numbering system equivalent to that used in the remainder of the ship, as negative numbers forward of the FP and numbers in excess of 10 (or 20, etc.) abaft the AP.

Body plan station planes are not to be confused with planes at which the vessel's frames are located, although frames are normally located in planes normal to the baseplane and longitudinal centerplane, which are therefore parallel to body plan station planes. Frames are normally spaced to suit the structure and arrangement of internals and their location is not dependent upon station plane locations. On some naval ships, frame spacing is an integral number of feet or one meter. Frame locations are usually chosen early in the design of a ship, and it is customary to show them on arrangement drawings and frequently also on final lines drawings. Therefore, frame locations, and their spacing, must be clearly stated. A body plan at frame locations is frequently drawn to assist the shipyard in fabricating the frames.

Frames are generally numbered with integer numbers, either starting at the FP and increasing aft, or at the AP and increasing forward. The latter practice is customary in tankers. In some instances, particularly naval ships, frames have been identified by the distance of the frame plane in meters from the FP.

A frame plane establishes a molded line, or surface, which will be coincident with the plane of either the forward or after edge of the frame. The location of frames, either forward of or abaft the frame line, should be clearly stated on relevant drawings.

The outline of the ship is completed in the sheer plan by showing the line of the main deck at the side of the ship, and also at the longitudinal centerline plane whenever, as is usual, the deck surface is crowned or *cambered*, i.e., curved in an athwartship direction with convex surface upwards, or sloped by straight lines to a low point at the deck edge. A ship's deck is also usually given longitudinal sheer; i.e., it is curved upwards towards the ends, usually more at the bow than at the stern. In case the sheer line of the deck at side curves downward at the ends, the ship is said to have reverse sheer.

Similarly, lines are shown for the forecastle, bridge, and poop decks when these are fitted; sometimes decks below the main deck are also shown. All such deck lines generally designate the molded surface of the

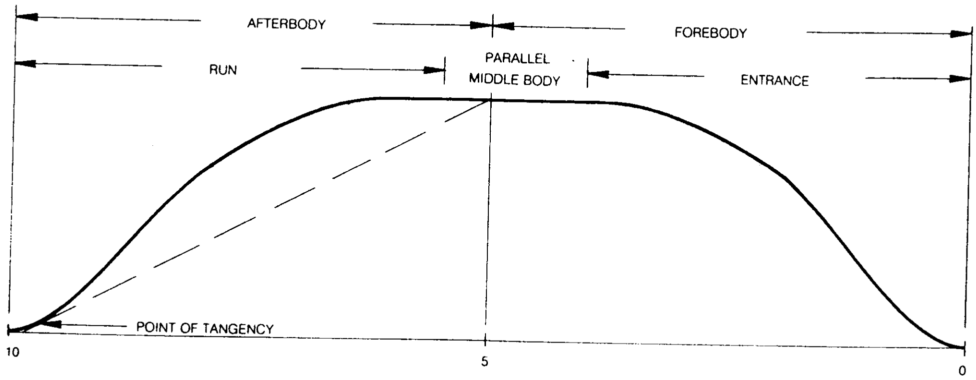


Fig. 3 Geometry of sectional area curve

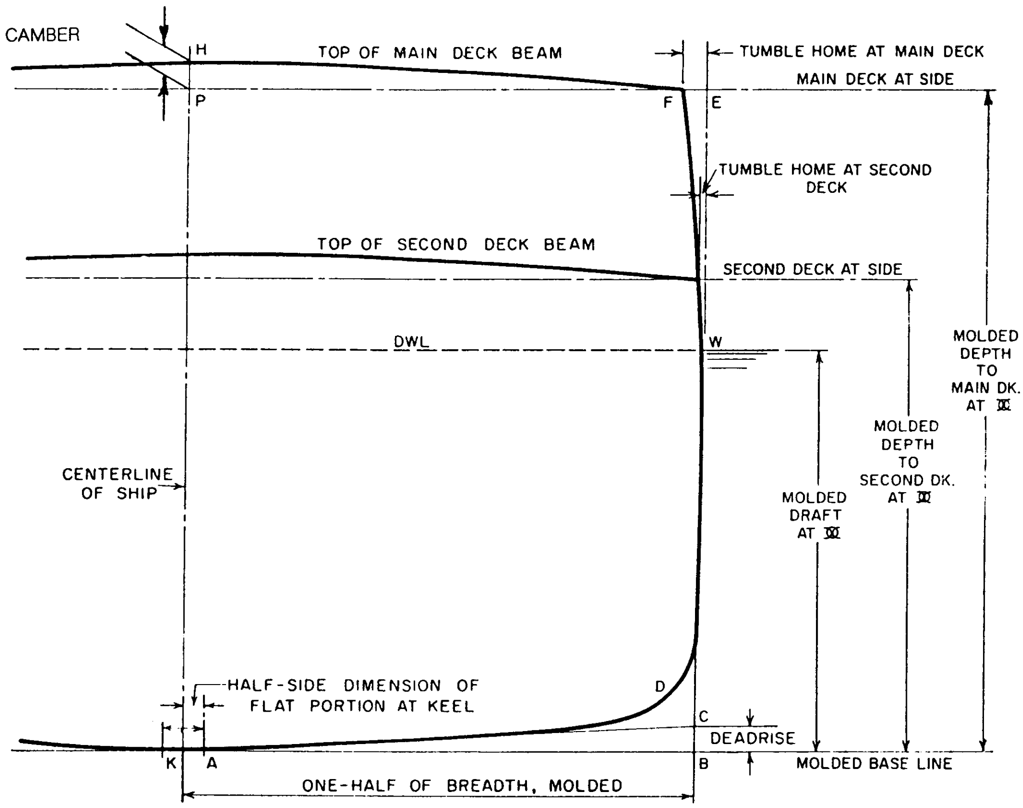


Fig. 4 Midship section, molded form

respective deck; i.e., the surface at the top of the deck beams, and are consequently referred to as molded deck lines at side or at center as the case may be.

In the lines drawing, Fig. 1, the curve of the main deck at the side is projected into the sheer plan as the curve $C'F'J'$ and into the body plan as $J''D''F'''$ for the fore body and as $F'''E''C''$ for the after body, and also into the half-breadth plan as the curve $C_1F_1J_1$, which is known as the half-breadth line of the main deck.

Through the point where the molded sheer line of the main deck at side intersects the midship station in the sheer plan, there may be drawn a level line called the molded depth line of main deck at side. At any particular station, the vertical distance between this line and the sheer line of deck at side is known as the *sheer* of the deck at that station. The sheer of a deck would, therefore, be zero at the midship station, and it may be zero for an appreciable distance either forward of or abaft amidships. Of particular interest are the values of sheer at FP and AP.

The sheer line of deck at side in some vessels, particularly yachts, may dip below the level of the molded depth line at side. This usually occurs, if at all, in the region immediately abaft amidships, and the sheer of the deck in such a region is measured below the level of the molded depth line at side and is considered to have a negative value.

The molded lines of the principal transverse bulkheads are sometimes also shown on final drawings.

1.5 Molded Base Line; Molded Dimensions. The molded base line, drawn in the sheer plan and body plan as a straight horizontal line, represents an important reference datum, both for design and construction purposes. The line, in fact, represents a plane in space to which many vertical heights are referred. It also represents the bottom of the vessel's molded surface, and so is coincident with the top surface of the flat plate keel on most straight-keel ships with a single thickness of shell plating.

In the event the keel line of a ship is straight, but the vessel has a designed drag to the keel, it usually slopes downward aft. In this case the molded base line may mark the bottom of the molded surface amidships, or at the AP. When drawing the lines for such a vessel, the bottom of the molded surface is shown as a raked line.

In the event the vessel is designed with an external hanging bar keel, extending below the shell plating surface, the bottom of keel is drawn in the sheer plan to complete the lower contour of the vessel. However, on most other ships, only the bottom of the molded surface is drawn.

In the case of ships with "in and out" riveted plating, the keel plate is usually an "out" strake and the bottom of keel is then below the molded base line by not only its own thickness but that of the first outboard, or garboard strake, as well.

The molded depth of a vessel is the vertical distance between the molded base line and the molded depth line of the uppermost deck at side as shown in Fig. 4.

The distance from K to B in Fig. 4 is one-half of the important dimension known as the molded beam or molded breadth of the vessel, which is normally a maximum at the midship station.

1.6 Characteristics of the Sections. In Fig. 4 from the point A the molded line of the bottom of the midship section extends towards the side in a straight line AC. This line often is inclined upwards slightly and intersects, at the point C, the vertical line EB drawn tangent to the widest part of the underwater body.

The line AC is known as the floor line, and the distance $\overline{BC}$ is referred to variously as the *deadrise*, rise of floor, or rise of bottom. For the ship shown in Fig. 1, the deadrise is 0.305 m.

The point K in Fig. 4 at the vessel's centerline is at the lowest part of the molded surface and the distance $\overline{KA}$ is the *half-side* dimension of the flat portion of the molded surface in the vicinity of the keel i.e., to the beginning of the deadrise. This half-side dimension is small in vessels having a hanging bar keel, being simply the half-thickness of the bar forming the keel, but in vessels having a dished-plate keel it will be considerably more, depending upon the size of the ship. It does not apply at all to ships with no deadrise.

The curved portion of the section, as at D, which joins the floor line with the side, is known as the *turn of bilge* and may be further described as a "hard" or as an "easy" turn of bilge, where hard refers to a small radius of curvature. The turn of bilge throughout the parallel middle body is usually, but not necessarily, a circular arc, and the radius of this curve is known as the *bilge radius.*

The molded line of the side above the waterline sometimes extends inboard somewhat to meet the line of the top of the main deck beam. In Fig. 4 this intersection is at the point F. The horizontal distance $\overline{EF}$ is known as *tumble home* at the deck. The opposite of tumble home is known as *flare*, and it is measured in a similar way.

A horizontal line through F in Fig. 4 meets the centerline of the section at P; the distance PH is called *camber* or *round of beam.* The camber curve may be an arc of a circle, a parabola, or several straight lines. Standard past practice has been to provide about 2 percent of the total breadth of the ship as camber amidships, and then to use the camber curve so determined as applicable to all other fore and aft locations. The use of camber accomplishes the important function of assuring that rain water and water shipped aboard will drain off readily.

1.7 Sectional Area Curve. A fundamental drawing in the design of a ship—particularly relative to resistance—is the sectional area curve, shown in Fig. 3 for a ship with some parallel middle body. The sectional area curve represents the longitudinal distribution of cross sectional area below the DWL. The ordinates of a sectional area curve are plotted in distance-squared units. Inasmuch as the horizontal scale, or abscissa, of Fig. 3 represents longitudinal distances along the ship, it is clear that the area under the curve represents the volume of water displaced by the vessel up to the DWL, or volume of displacement.

Alternatively, the ordinate and abscissa of the curve may be made non-dimensional by dividing by the midship area and length of ship, respectively. In either case, the shape of the sectional area curve determines the relative "fullness" of the ship (See Section 3).

The presence of parallel middle body is manifested by that portion of the sectional area curve parallel to the baseline of the curve. The shoulder is defined as the region of generally greater curvature (smaller radius of curvature) where the middle body portion of the curve joins the inward sloping portions at bow or stern.

The centroid of the vessel's sectional area curve is at the same longitudinal location as the center of buoyancy, LCB, and the ratio of the area under the sectional area curve to the area of a circumscribing rectangle is equal to the *prismatic coefficient*, C_p (See Section 3).

Fig. 3 also shows the customary division of the underwater body into *forebody* and *afterbody*, forward of and abaft amidships, respectively. *Entrance* and *run*, which represent the ends of the vessel forward of and abaft the parallel middle body, are also shown.

1.8 Molded Drafts; Keel Drafts; Navigational Drafts; Draft Marks. In general, the amount of water a vessel draws, or *draft*, is the distance measured vertically from the waterline at which the vessel is floating to its bottom. Drafts may be measured at different locations along the length. They are known as molded drafts if measured to the molded baseline; keel drafts if measured to the bottom of the keel. Mean draft is defined as the average of drafts forward and aft.

Ships are customarily provided with draft marks at the ends and amidships, arranged in a plane parallel to station planes and placed as close to the perpendiculars as practical. These draft marks are for the guidance of operating personnel, and therefore the drafts indicated should be keel drafts. The marks are painted in a readily visible color to contrast with the color of the hull. Arabic numerals are usually used on merchant vessels, although Roman numerals also appear on some naval ships, particularly in way of appendages that extend below the baseline. The bottom of the numeral is located at the indicated waterline. For many years it has been the practice to use numerals 6 inches high and to mark the drafts in feet at every foot above the keel. Thus, if one were to see the numeral half immersed, the prevailing draft would be three inches deeper than the half-immersed number in ft.

With the ultimate conversion to the metric system in the United States a reasonable practice would seem to be that adopted by Australian maritime authorities (Australian Dept. of Transport, 1974). This provides that drafts be shown in meters at every meter in Arabic numerals, followed by M. Intermediate drafts are shown at every 0.2 m (2 decimeters), but only the numerals 2, 4, 6 and 8 are shown, with no decimeter designation. All numerals are to be one decimeter high. Thus, draft marks between 11 and 12 meters would show,

12M
8
6
4
2
11M

The difference between drafts forward and aft is called *trim.* If the draft aft exceeds that forward, the vessel is said to have trim by the stern. An excess of draft forward causes trim by the bow—or trim by the head. When trim is determined by reading the draft marks and the angle of inclination or the displacement of the vessel is to be determined, it is important to account for the specific fore and aft location of the marks.

Some vessels are designed with local projections below the keel of a permanent nature—for example, sonar transducer housings (domes), and the propeller blade tips of some naval vessels. It is important that operating personnel be well aware of the distance below the keel to which such projections extend. Navigational drafts—which represent the minimum depth of water in which the vessel can float without striking the bottom—would exceed keel drafts by this distance.

1.9 Diagonals; Types of Intersecting Planes. The shape of curves shown by the stations, buttock lines and waterlines do not necessarily convey the shape of hull form as one might wish to see it, and the designer need not be limited to use of these planes. Additional planes with which the hull form is sometimes intercepted are diagonal planes, which are planes normal to station planes, but inclined with respect to the baseplane and the longitudinal centerplane. Such a plane appears as a straight line in the body plan. The inclination of a diagonal plane is generally chosen so that it is approximately normal to the body plan sections.

It is customary to show the resulting intercept curve, called a *diagonal,* below the half-breadth view in the lines drawing. This practice has been followed in Fig. 1. Thus, the expansion of the diagonal is a plot of distance from the point *W* on the ship's C.L. in the body plan to the points where *ZW* crosses each station. The particular diagonal shown in Fig. 1 is called a *bilge diagonal,* inasmuch as it intersects the bilge. Point *W* is at the DWL on the vessel's centerline, and point *Z* marks the intersection of the vessel's half-beam line and deadrise line.

Projections showing the intersections of diagonal planes with the molded surface are generally omitted in the half-breadth plan and the profile.

1.10 Cant Frame Lines. On some types of vessels, it is found that near the ends of the vessel, the inclination of the ship's surface to the planes of transverse frames becomes so great as to require these planes to be moved to a position more nearly normal to the surface, so that the frame when so constructed may give a better support to the surface in its vicinity. In the event the plane of the frame remains normal to the baseplane, the trace of the plane in the centerplane appears as a line perpendicular to the molded baseline in the sheer plan, and the frame is called a *single cant frame,* or simply *cant frame.*

Frames are also occasionally placed in planes normal to the longitudinal centerplane, but inclined to the baseplane, whereby the trace of the plane in the baseplane, as seen in the half-breadth plan, is perpendicular to the vessels' centerline. The term *inclined frame* has been applied to this case.

Double cant frames lie in planes which are neither normal to the longitudinal centerplane nor the baseplane. Determining the trace of such a plane in the molded surface is an exercise in descriptive geometry, and such frames are rarely used.

1.11 Fairness: Fairing of Lines. It is of interest to note certain features of hull form shown by the lines in Fig. 1. The lower waterline shapes near the bow and stern are drawn with some *hollow*—that is, they are concave. Similarly, the body plan sections and buttocks are hollow, generally, in the vicinity of the DWL and particularly in the afterbody. Thus the shape of the lines in one view is reflected in the other views, and vice versa.

With the exception of deliberate discontinuities at the stem, knuckles, chines, transom corners, etc., the shape of a vessel's exterior form below the deck is virtually always designed as a fair surface. A fair surface is defined as one that is smooth and continuous, and which has no local bumps or hollows, no hard spots and a minimum of points of inflection. Localized flat spots between areas of the surface with curvatures of equal sign are generally considered unfair, unless they occur as part of the bottom or sides, especially with parallel middle body. Mathematically, the property of fairness of surface might be thought of as that of continuity in a plot of curvature, or radius of curvature, of the intersection of any plane with the surface. Inasmuch as waterlines, buttocks, station lines and diagonals all represent the intersection of planes with the molded surface, it may be seen that a fair hull form will be characterized by fairness in these curves; correspondingly, it is usually assumed that if these curves are fair, then so will the hull form. In general, discontinuities in the first derivative, indicating abrupt changes in slope, occur at knuckle lines. Other sudden changes in curvature, indicated by discontinuities in the second derivative, are considered to show unfairness. A common situation on ships with parallel middle body is a bilge of constant radius, r, connecting to flat bottom and/or side, with a change in curvature of the transverse section from $1/r$ to 0 at the point of tangency. Although such a section is not fair, its shape is not necessarily disadvantageous. It can be made fair if desired by easing the transition in curvature. On the other hand, continuity in both first and second derivatives does not guarantee fairness, inasmuch as the achievement of fairness has always been and probably will continue to be a matter of opinion or judgment.

An additional condition implied by the term fairness is that of consistency, that is, each projection of any point on the surface onto the corresponding reference plane must agree with the locations of its other projections. For example, consider a point P to be on the surface of the ship in Fig. 1 at station 7 and 4 ft (1.22 m) above the molded base line. This point would be shown in the sheer plan at P'. Its location in the body plan would be on transverse section 7, and on the 4-ft WL. The horizontal distance of the point P from the ship's centerplane would be determined by the distance in the body plan of the point P'' from the ship's centerline, as $\overline{P''R''}$. The point P_1 in the half-breadth plan would be at the ordinate for station 7 and on the 4-ft WL and its distance from the ship's centerline would be $\overline{P_1R_1}$ as shown in that plan. A test of consistency of the point P would be that the distance $\overline{P_1R_1}$ in the half-breadth plan must equal $\overline{P''R''}$ in the body plan.

In case the point P had been originally selected on the surface at a location where no transverse section, waterline, or buttock already existed, a check of fairness would require one to introduce any two of these three types of intersecting planes through the point, find the corresponding projections of the lines of intersection and proceed as before.

The process of fairing a set of lines is invariably an iterative, or cut and try one, requiring patience and perseverance. It consists essentially of investigating the fairness or suitability of each line of the vessel in succession. It often happens that, after testing and accepting a number of lines, the next line to be considered will require changes to be made to it that will be so far-reaching as to affect some of the lines previously accepted. It then becomes necessary to make whatever changes seems best, all things considered, and to proceed anew through the same fairing steps as before. Usually several such difficulties have to be overcome successively before the whole fairing process is completed. Thus, the process may be laborious.

Fairing lines for a new ship design is normally accomplished at least twice—first in the design phase, and second in the construction phase, at which time the lines are faired either full-scale, on the mold loft floor, or in the optical detailing room to a scale of 1/10 or 1/20 of full size, or by computer as discussed in Section 1.16. In the design phase, there is greater freedom to make changes and to achieve hull form features which the designer favors. Curves are usually drawn using a combination of free hand sketching, ship curves and flexible battens (or splines) held by batten weights ("ducks").

Waterlines are usually drawn by the last of these methods. A uniform batten will be fair between a pair of ducks, but it can be forced into an unfair overall curve by the ducks. Hence, a customary method of fairing or smoothing is to adjust the ducks—and hence the batten defining the waterline—until any one of the ducks can be removed without the batten moving. This is intended to assure that changes in curvature are made gradually.

In the final design or construction phase, the lines are reasonably well defined at the start. The process of fairing is more localized and directed at achieving consistency among the various views. However, the larger scale used in this case is intended to assure that local deviations, which may not have been evident in the earlier small-scale design phase, will be eliminated.

1.12 Developing a Set of Lines. The development of a set of lines presupposes a tentative (or final) selection of suitable hull dimensions, coefficients (section 3), LCB, sectional area curve (Fig. 3) and design waterline. This selection is based on considerations of displacement, capacity, trim, stability, resistance and propulsion, all of which are discussed in other chapters, as well as in the chapter on Mission Analysis and Basic Design, *Ship Design and Construction* (Taggart, 1980). Fig. 5 is a generalized plot whereby the offsets of a sectional area curve may be drawn to fit prescribed hull features (prismatic coefficients and LCB.) In order to use Fig. 5, one enters Fig. 5a with LCB and total C_P to get C_{PA} and C_{PF}; these are then used in Fig. 5b to find the sectional area curve offsets.

Given the desired hull characteristics, the process of drawing and fairing a preliminary small-scale set of lines generally begins with fixing the profile of the vessel in the centerplane, the design waterline and deck line in the half-breadth plan, and the midship body plan section. Intermediate sections may next be sketched in to satisfy the pre-determined sectional area curve, often by reference to previous designs and typical hull forms (SNAME Hydrodynamics Committee, 1966). A few additional waterlines, between the deck and the DWL, and between the DWL and the baseline, are then drawn in the half-breadth view using half-breadths at the stations and making as small and as few changes as possible in these. The sections in the body plan are then changed to achieve consistency with the waterline half-breadths, and section areas checked. A few buttocks are then drawn in and checked and the process repeated. Alternatively, diagonals, rather than waterlines, are preferred by some designers as a fairing medium, and are used to check the consistency of section shape variation from station to station before buttocks and intermediate waterlines are drawn. Liberal use of the eraser is required, the drawing frequently being made on the back of transparent cross-section paper, chosen so that the grid of the paper matches the grid of waterlines, buttocks and

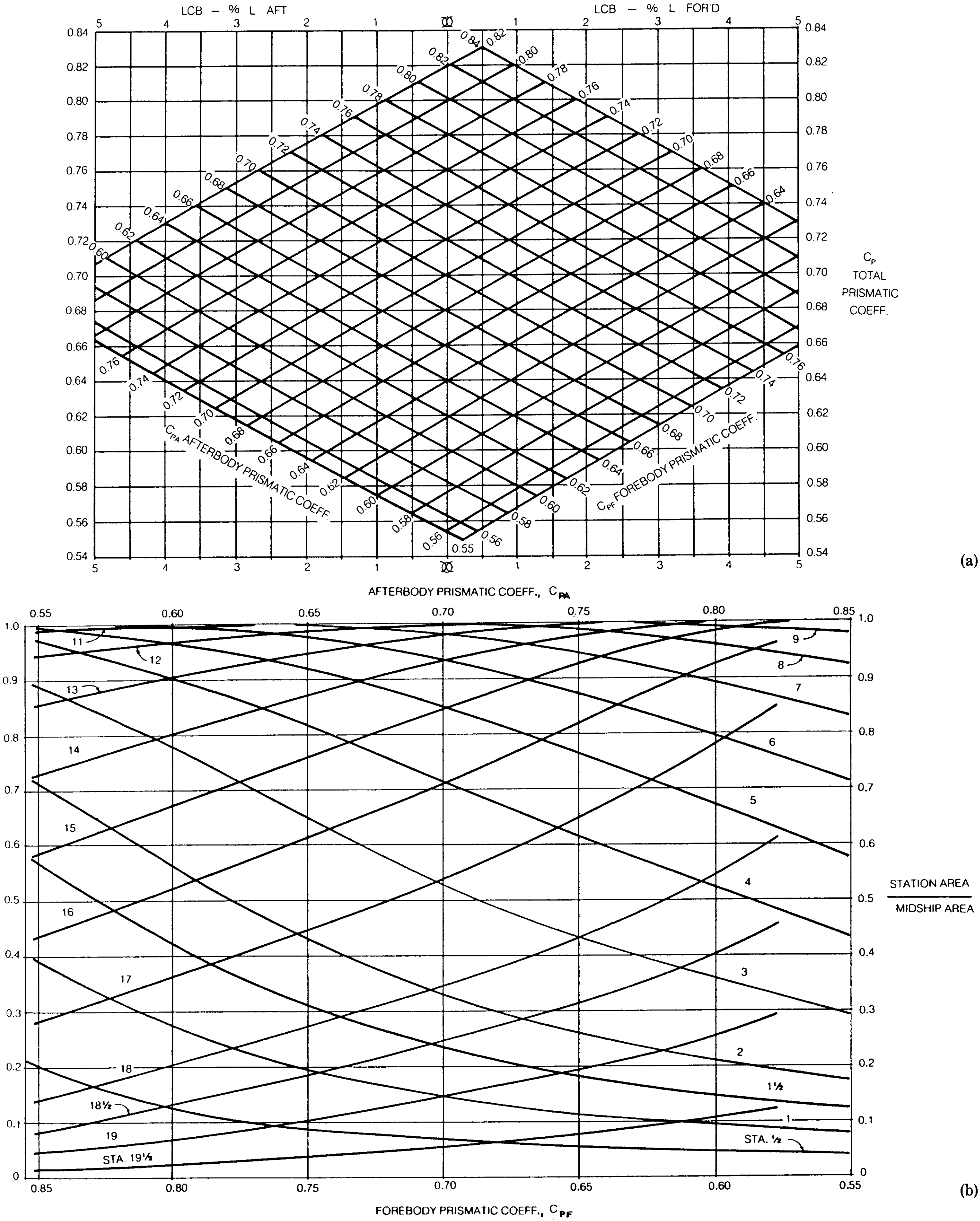


Fig. 5 Generalized plot of sectional areas, including forebody and afterbody prismatic coefficient as functions of longitudinal center of buoyancy (a & b)

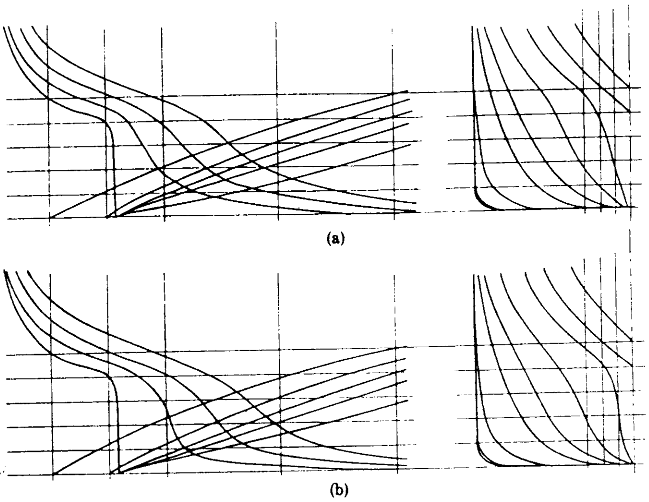


Fig. 6 Typical original and modified stern lines

stations desired. Because of the flatness of angle of intersection of buttocks and waterlines on narrow, fine-lined ships at the quarter length, it is sometimes the practice to foreshorten station spacing in the profile and half breadth plan to assist in fairing.

As the ship design progresses, one or more larger scale lines drawings must be prepared and faired with increasing precision. In the fairing process, some general guides should be remembered. For example, the general shape of buttock endings, particularly for buttocks near the ship's centerline, must reflect the shape of body plan stations, if a gradual and progressive hange of waterline slope is to be achieved. Fig. 6A illustrates a case in which (a) this guide was not followed, and (b) the lines might be modified to suit the guide.

Nine of the often conflicting considerations involved in developing a set of lines, other than those of resistance and propulsion discussed in detail in Chapter V, may be noted here:

(a) Generous clearances around the propeller tend to reduce vibration excitation forces, but a large diameter propeller tends to improve populsive efficiency and hence to reduce required shaft horsepower, assuming the propeller design is not restricted in RPM.

(b) A large amount of "fin" area aft, both fixed and movable, tends to promote directional stability. Generous movable area (rudder area) tends to improve the ability to initiate and recover from turns.

(c) A small bilge radius, together with a bilge keel right at the turn of the bilge, tends to increase roll damping. However, wetted surface, and hence frictional resistance, tend to be increased by a small bilge radius.

(d) V-sections are generally favorable to stability and seakeeping performance, but are often objectionable from the viewpoint of resistance and/or propulsion.

(e) Ships which must operate in heavy weather may experience slamming on the flat of bottom forward unless large deadrise angles are used and the extent of flatness is minimized. However, a long straight flat keel is desirable from drydocking considerations.

(f) Generous flare forward, sometimes with a gently sloped longitudinal knuckle well above the waterline, may be used instead of an increase in freeboard forward in achieving dry decks when in a seaway.

(g) Ships with bulbous bows may experience damage to the bulb from anchor handling unless the bow in way of the hawsepipe is flared out sufficiently to allow an unobstructed drop from the pipe extremity, taking into account the possibility of the ship's rolling to the opposite side.

(h) Hull surfaces composed of portions of cylinders and cones—i.e., developable surfaces—are more easily fabricated than surfaces of compound curvature, but may incur added resistance. See section 1.14.

(i) Excessive waterline angles forward of the propeller should be avoided, as well as blunt waterline endings, since they may promote separation in the flow, especially in the case of very full, slow-speed vessels. Such separation tends to cause propeller-excited vibration, as well as greater resistance and less efficient propulsion.

1.13 Offsets. In the process of building a ship, some means must be devised for determining the shapes of the frames with greater precision than can be obtained directly from the usual lines drawings. It has been the practice in most shipyards in the past, to attain the necessary accuracy, to redraw and refair the lines to full size on a large wooden floor located in a space known as the mold loft. The mold loftsmen were supplied sufficient information to enable certain portions or the whole of the vessel's lines to be drawn full size, often in contracted form, i.e., with all breadths and heights full size but with lengths reduced. These operations, complete with refairing as necessary, are known as laying off or laying down the lines.

For laying off a mold loftsman needs not only the lines drawing but also a list of the measurements he must use in locating points through which the various curves are to be drawn. Consider a waterline in the half-breadth plan and suppose that the distance on each station from the vessel's centerline to the waterline were measured. Such measurements are known as *offsets*, and by their use the loftsman can lay off the necessary points on the floor through which the required curve can be drawn in a fair line by using long flexible wooden battens. For a buttock line in the sheer plan, the offsets would be given as heights above the molded base line at each station. If in the English system, these would be in feet, inches, and eighths (or sixteenths) of an inch. It is expected that as shipyards in the U. S. convert to the metric system, full-scale offsets will be recorded to the nearest millimeter—that is, three decimal places after the meter—inas-

Table 1—Typical Table of Offsets

Station	Halfbreadths, m: Half Siding	Bottom tangent	4-ft WL 1.219 m	8-ft WL 2.438 m	16-ft WL 4.877 m	24-ft WL 7.315 m	27-ft WL 8.230 m	32-ft WL 9.754 m	Station
0, FP	0	—	0.759	0.581	0.108	—	—	0.133	0, FP
½	0.394	—	1.308	1.432	1.270	1.172	1.245	1.613	½
1	0.483	—	1.968	2.438	2.730	2.962	3.140	3.610	1
1½	0.571	—	2.978	3.848	4.626	5.102	5.359	5.886	1½
2	0.660	—	4.324	5.534	6.575	7.315	7.597	8.093	2
3	"	0.860	7.509	8.909	10.173	10.792	10.956	11.195	3
4	"	3.832	10.293	11.208	11.830	11.986	12.007	12.033	4
5	"	9.144	11.417	11.916	12.039	12.039	12.039	12.039	5
6	"	6.268	10.344	11.338	11.983	12.039	12.039	12.039	6
7	"	2.324	6.833	8.490	10.627	11.703	11.899	12.033	7
8	"	0.679	3.314	4.423	6.788	9.458	10.271	11.246	8
8½	"	0.660	2.207	2.896	4.518	7.306	8.417	9.976	8½
9	0.660	—	1.445	1.778	2.508	4.677	5.962	7.973	9
9½	0.432	—	0.549	0.568	0.600	1.553	3.057	5.410	9½
10, AP	—	—	—	—	—	—	—	2.130	10, AP
10-ft aft (3.048m)	—	—	—	—	—	—	—	—	10-ft aft (3.048m)

Table 1 (continued)

Station	Halfbreadths, m: 40-ft WL 12.192 m	Main Deck	Foc'sle Deck	Buttock Heights, m: I 4-ft 1.219 m	II 8-ft 2.438 m	III 16-ft 4.877 m	IV 24-ft 7.315 m	V 32-ft 9.754 m	Station
0, FP	0.879	2.337	4.477	12.872	14.967	—	—	—	0, FP
½	2.775	4.483	6.674	0.911	11.586	15.335	—	—	½
1	4.823	6.518	8.477	0.378	2.438	12.284	15.888	—	1
1½	6.988	8.404	9.934	0.178	0.787	6.401	12.808	17.066	1½
2	8.979	9.966	11.011	0.083	0.368	1.654	7.315	14.167	2
3	11.484	11.716	11.944	0.019	0.089	0.359	1.111	3.810	3
4	12.039	12.039	—	0.016	0.048	0.117	0.251	0.851	4
5	"	"	—	"	"	0.111	0.178	0.279	5
6	"	"	—	"	"	0.111	0.213	0.835	6
7	12.039	"	—	0.016	0.048	0.394	1.524	3.708	7
8	11.932	12.039	—	0.044	0.517	2.953	5.347	7.630	8
8½	11.389	11.890	—	0.175	1.600	5.264	7.325	9.503	8½
9	10.252	11.370	—	0.676	4.712	7.461	9.223	11.510	9
9½	8.236	10.001	—	7.074	7.852	9.389	11.271	14.478	9½
10, AP	4.861	6.826	—	9.093	9.989	12.211	—	—	10, AP
10-ft aft (3.048m)	2.658	4.553	—	10.598	11.919	—	—	—	10-ft aft (3.048m)

much as one millimeter = 0.0397 in. ≈ 1/25 in., very nearly.

A complete set of offsets for the various lines of the vessel, arranged in tabular form, is known as a table of offsets. The typical example given in Table 1 applies to the ship shown in Fig. 1. Sometimes such tables are included on the lines plan. The offsets originally supplied to the loftsmen are usually marked "preliminary." After the lines have been faired on the loft floor, another set of offsets, known as the "returned" or "finished" table of offsets is usually lifted from the floor and returned to the drafting office. This finished set should include offsets lifted for every frame station throughout the ship, in addition to those for the lines stations.

Fairing ship lines on a mold loft floor is time consuming and requires substantial amounts of floor space. To overcome these disadvantages, some shipyards in the 1950s began to have the preliminary lines redrawn and refaired to a scale of one-tenth of full size, the work being done on large drawing tables with precise drafting instruments. Originally 1/10-scale drawings of structural parts were made and photographically reduced to 1/100 or less of full scale. The photographic negatives were then projected optically full-size onto the plates for marking and cutting. Later

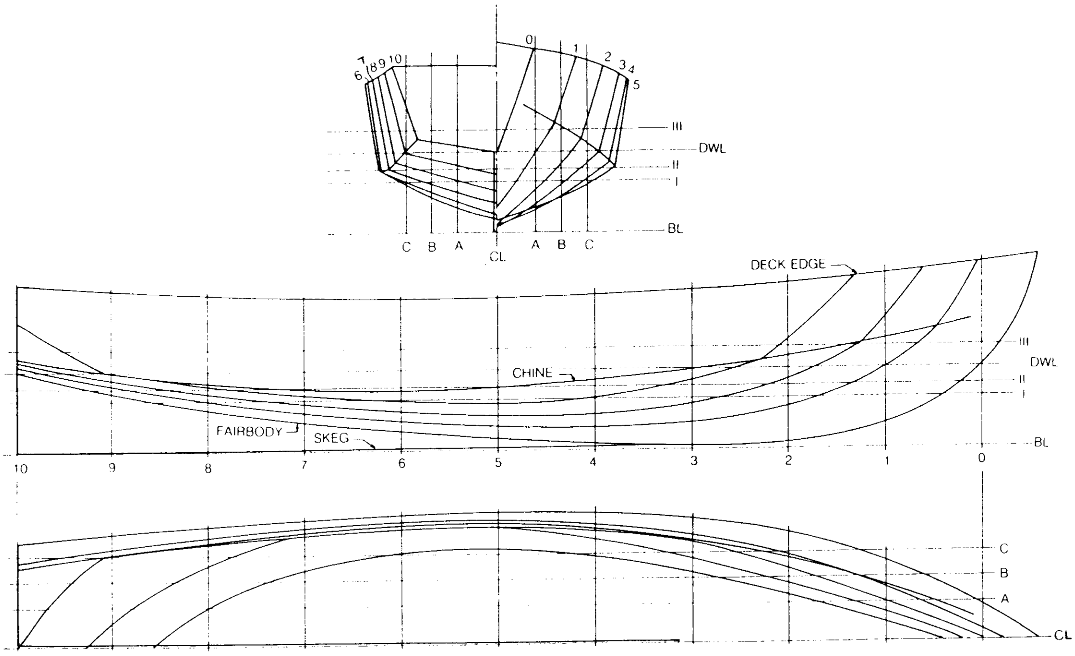


Fig. 7 Lines of small developable surface vessel

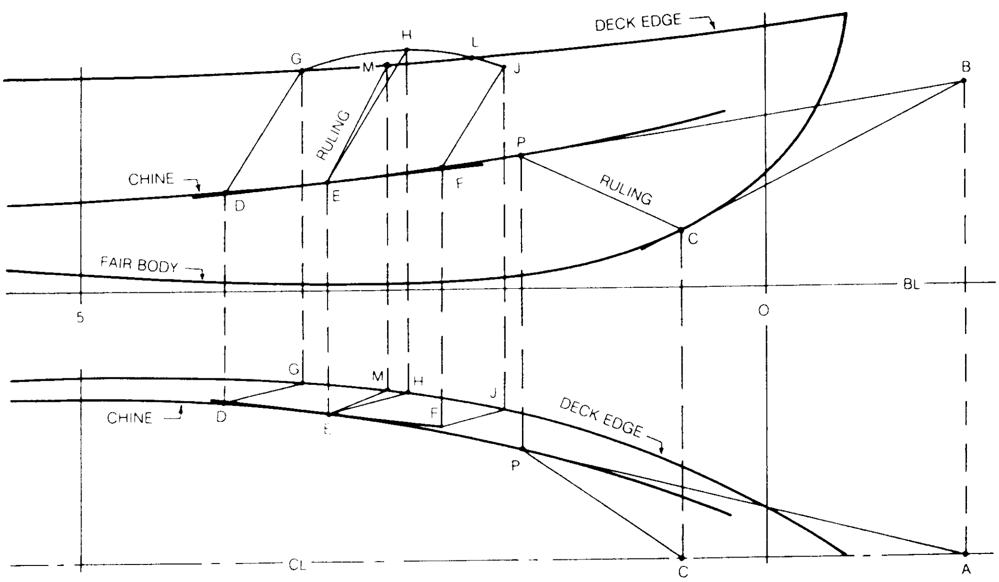


Fig. 8 Construction drawing for developable surface lines

To find ruling between deck edge and chine:

1. Draw $\overline{DF}$, tangent to chine at E.
2. Draw $\overline{DG}$, $\overline{EH}$ and $\overline{FJ}$ in half-breadth plan parallel to each other at arbitrary angle.
3. Project G to deck edge in profile.
4. Find points M and J in profile as projections from half-breadth where $\overline{DG}$, $\overline{EH}$ and $\overline{FJ}$ are parallel.
5. Draw curve GHJ in profile cutting deck edge at L.
6. M is midpoint of GL and is end of ruling $\overline{EM}$
   Plane DGJF is tangent to chine at E.
   Curve GHLJ is the intersection of plane DGJF with a cylindical surface with vertical elements through deck edge.

To find ruling between stem profile and chine:

1. Draw $\overline{PA}$ and $\overline{PB}$ tangent to chine at P.
2. Project point A in half breadth at CL, to point B in profile.
3. Draw $\overline{BC}$ tangent to stem profile at C, giving end C of ruling $\overline{PC}$.

the 1/10-scale drawings were used directly as templates for optically controlled burning machines. Offsets from the 1/10-scale lines were considered to be the finished offsets.

During the 1960s and 1970s there was rapid development of the use of computers, and most large shipyards now use computers as an aid to the entire process of fairing the lines (section 1.16), computing the offsets, and preparing numerical control for automatic flame cutting. See Chapter XVI, *Ship Design and Construction* (Taggart, 1980) for further details.

1.14 Developable and Straight-Frame Lines. Ship hull forms as traditionally designed are composed of surfaces of compound curvature, such that the intersection of any plane with the surface will form a curved line. A simpler type of curvilinear surface is one on which the intersection of certain planes with the surface form straight lines—called *rulings* or *elements*—which never cross each other. Such a surface is known as a developable surface because it is possible to unbend or unroll the surface and flatten it into a plane. Hence, it will usually be a portion of cylinders or of cones. Correspondingly, it is possible to form a developable surface from a plane surface, such as a sheet of paper or a sheet of steel, by bending it in only one direction along successive rulings.

Hull forms composed entirely of developable surfaces have been successfully designed, particularly for smaller vessels, but they have potential value for larger vessels as well. The different surfaces usually connect at *chines*, or curved knuckle lines, which should be oriented to follow lines of flow as much as possible.

According to the method described by Kilgore (1967), a developable surface can be formed to include two arbitrary curves in space, provided the curvature of the projections of the two curves on the planes of a Cartesian system always have the same sign. However, this is not a necessary condition and the drafter must rely on experience and sometimes on trial. Michelsen, in discussing Kilgore (1967), gives methods by which the existence of a developable surface between two space curves may be checked.

Chine and deck edge lines usually meet the constant curvature sign condition. When they do not, test construction lines may be drawn according to the method outlined to see if rulings can be determined. If none are found, one or both of the curves may be modified and rechecked. In general, it may be said that straight line stems and points of inflection in the chine and deck edge should be avoided. Points of inflection should also be avoided in the intersection of the bottom surface of the vessel with the longitudinal centerplane, defined as *fairbody* line (Kilgore, 1967).

Fig. 7 shows the lines for a small single-chine developable surface vessel. It will be seen that body plan stations, especially forward between the fairbody line and the chine, are slightly convex when seen from the exterior, which is characteristic of developable surface lines in general.

The construction drawing, Fig. 8, shows how rulings on the side between deck edge and chine may be drawn. A ruling is not only a straight line element of the developable surface, but also lies in a plane tangent to it. The construction at the bow shows how rulings are found between the stem profile and the chine. Rulings in the bottom surface may be found in a manner analagous to that used between the chine and deck edge. Kilgore (1967) provides the basic theorems which govern the determination of rulings and the uniqueness of the resulting developable surface. A useful feature of rulings is that tangents to buttocks in the profile view at a single ruling are parallel to each other, and tangents to waterlines in the halfbreadth plan at a single ruling are parallel to each other.

The process of drawing lines for a developable hull form is, therefore, one of finding rulings between chine and deck edge, between pairs of chines, and between chine and fairbody line. Once the rulings are found, it is a relatively simple task to find stations, waterlines and buttocks using normal projection techniques, inasmuch as the rulings are defined in two views.

Fig. 9 shows the body plan of a comparable straight-frame vessel and it may be seen that differences, compared with the developable surface vessel, are relatively small. Thus, the choice of designing a vessel with one system or the other may well depend upon comparing the difficulty of forming curved frames—together with the ease of plating the developable shell—with the ease of forming straight frames—together with the difficulty of plating a warped shell.

1.15 Methodical Lines. If the naval architect wishes to draw lines representing a particular type of ship, there are in the open literature several *methodical series* of hull forms which permit offsets to be developed directly, without the necessity of going through the fairing process. By a methodical series is

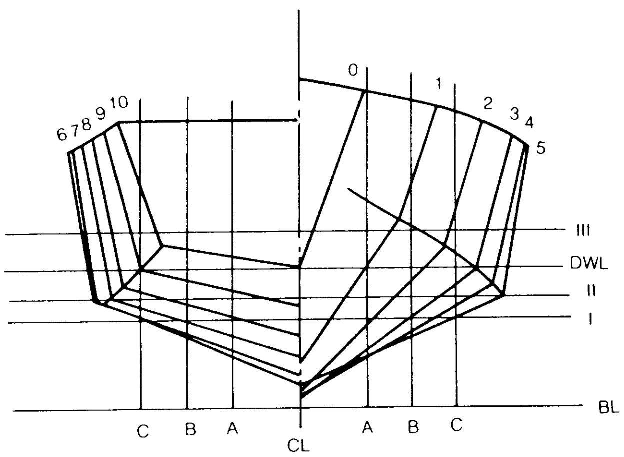


Fig. 9 Straight frame body plan of vessel

meant a group of uniquely related forms whereby specific offsets may be obtained from charts or tables for given arbitrary input characteristics, especially prismatic coefficient, length-to-breadth ratio, volumetric coefficient, and position of the longitudinal center of buoyancy. Particular examples are the Series 60 single screw merchant ship forms (Todd, et al, 1957), Taylor's Standard Series (Taylor, 1943), Townsend's seakeeping series of single screw forms (Townsend, 1967), the MARAD low L/B series (1987), and the Webb trawler series (Ridgely-Nevitt, 1963). Furthermore, by following methods outlined in Section 3.3, the lines of any specific hull form may be transformed in a methodical way to suit arbitrary hull form characteristics.

The constraints of end details—stern frame ending, size and location of rudder, stem profile, etc.—with which such methodically chosen lines are endowed often leads to "tailoring" of the resulting lines at the ends, which may require refairing substantial portions of the hull surface.

1.16 Use of Computer in Lines Definition; Mathematical Lines. Among the most useful applications of digital computers in naval architecture, giving direct geometrical answers, are:

(a) Determining lines and offsets to suit arbitrary hull form characteristics derived from a prescribed parent form.

(b) Final fairing and determination of closely spaced frame offsets for shipyard use based upon widely spaced preliminary design offsets.

The first of these applications provides the capability to carry out by computer and in a more general fashion what Adm. D. W. Taylor began prior to World War I, that is, to design ship lines mathematically. By the method of Taylor (1915) waterlines and sectional area curves took the form of a 5th order curve, separately for forebody and afterbody,

$$y = tx + ax^2 + bx^3 + cx^4 + dx^5,$$

where t, a, b, c and d are constants.

By suitable transformation, this equation was rewritten as,

$$y = C_y + PC_p + tC_t + \alpha C_\alpha,$$

where P is the waterplane area coefficient (for a water line), or prismatic coefficient (for a sectional area curve) of forebody or afterbody, t is the tangent of the curve at bow or stern, and α is a function of the second derivative of the curve at amidships ($x = 1.0$). A simple table was provided giving values of the coefficients, C, which were fixed for each body plan station. The resulting curves had at most one point of inflection. This method was used to draw the lines for Taylor's Standard Series (of ship models used in resistance tests—see Chapter V). Taylor (1915) noted that, "practically all U.S. naval vessels designed during the last ten years have had mathematical lines."

During the intervening 65 years, the use of mathematical ship lines appears to have declined until the advent of computers. A number of successful attempts have now been reported (Fuller, et al, 1977 and Söding, et al, 1977, for example) where ship lines in keeping with hull forms favored today have been produced and plotted with the aid of the computer. Polynomials of higher order than used by Taylor have been used for waterlines and sectional area curves, with particular attention taken to avoid unwanted points of inflection. However, unless some adjustment is done to the end profiles, the resulting hull forms are endowed with waterline endings or stern profiles that may not satisfy the user. Kuiper (1970) presented a method whereby the design waterline is expressed as two eight-term polynomials, one for forebody and one for afterbody, which are easily determined using the basic hull form characteristics. However, to define the hull form above and below the design waterline requires the use of seventeen form parameters which must be defined at all drafts, for forebody and afterbody.

Recently somewhat different computer techniques have been developed to assist in the early stage of lines development. For example, a Ship Hull Form Generator Program (HULGEN) was developed in the Ship Design Division of NAVSEC. (Fuller, et al, 1977). The key to this program is the use of polynomials in various combinations to build up a line-for-line definition of the hull form that is remarkably fair. The strength of the program is the user-oriented interactive-graphics method of data input, display and modification. Results of variations of parameters can be viewed instantly, or the hull form can be stretched and distorted into shapes to maintain those parameters.

The second application is that of final fairing of preliminary lines, which necessarily embodies judgment, in that the drafter's eye and opinion ultimately determine fairness. In this process, the drafter, or the mold loftsman, is faced with the problem of passing a curve through a set of points, usually equally spaced along a reference axis, and satisfying himself that the curve is smooth, with a minimum number of points of inflection and with curvature varying in a gradual way. In order to achieve fairness, the curve may have to miss some of the points by small amounts. Also, for consistency, interesecting curves in other views which contain these points must be checked and adjusted.

As previously noted (1.11), battens or splines are commonly used in drawing such curves, with batten weights (ducks) positioned to hold the batten at or near the given points. Therefore, computerized representations of ship lines often make use of the equations for spline curves. The bending induced in the batten by the ducks is describable by the theory of bending of a simple weightless beam with concentrated loads or supports at a series of discrete points, corresponding to the points of duck restraint. It is shown in Strength of Materials texts that the deflection of such a beam is given by polynomials no higher than the

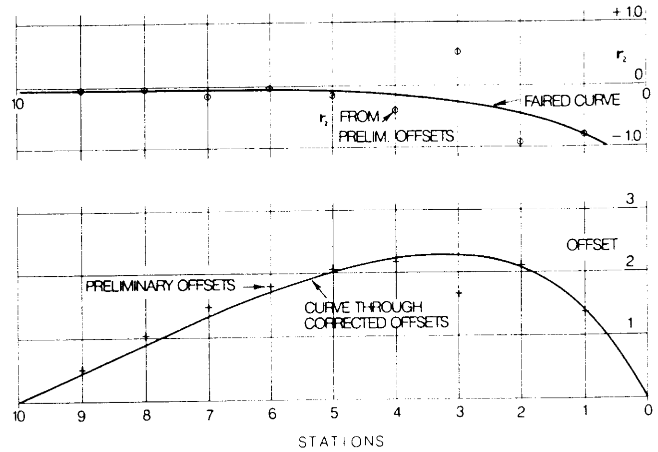


Fig. 10 Unfair and faired rudder section

third order, that is, by cubic functions of the x-dimension parallel with the beam. Such cubics have continuity in first and second derivatives ($y' = dy/dx$, $y'' = d^2y/dx^2$) at the points of application of the concentrated loads.

An assumption made in developing the deflection equation for a simple beam is that the deflections are small, and the beam assumes only small angles to the x-axis. Under these conditions the differential equation of bending is

$$M(x)/EI = y''/[1+(y')^2]^{3/2}.$$

This can be linearized by assuming that $(y')^2 = 0$, inasmuch as y' is small; thus,

$$M(x)/EI \approx y''$$

and y'' becomes a linear function of x. A close approximation to y'', at point n for example, is given by the second difference r_2, where

$$r_2 = (y_{n+1} - 2y_n + y_{n-1})/h^2,$$

and h is the spacing between any pair of equally spaced points. Inasmuch as r_2 is quite sensitive to changes in curvature, it is apparent that by adopting r_2 values from a smooth curve, and by adjusting offsets to match the faired r_2 values, a curve through the adjusted offsets will usually be quite fair.

For illustrative purposes, Fig. 10 plots rough—and obviously unfair—points representing preliminary offsets of a rudder section. Also shown is a plot of r_2 from the given offsets, and a smooth curve interpreting the plot but missing some of the points.

The final faired rudder profile curve has been obtained from the smooth curve of r_2, beginning at the nose of the section and working aft, but with the addition of two additional linear corrections, first to make the tail of the section sharp, and second, to make the average value of the faired offsets equal to the mean value of the given offsets.

The spline curve representation of ship lines by the differential equation of the deflection of a simple beam may become unrealistic when the slope y' of the line being represented becomes so large that it cannot be assumed equal to zero. Thus, most ships' waterlines can readily be represented by spline curve equations over most of the length of the ship. However, such is not the case for body plan stations, nor for many buttocks, especially near the ends of the ship, where steep slopes are often met. In order to overcome this problem, some early attempts to define ship lines with the aid of a computer required that the coordinate reference axes be rotated. More recently, lines have been expressed as parametric spline curves, by which the curves are defined by a parameter s, rather than directly by x,y coordinates. The parameter s is defined as the cumulative length of segments of the line from the start point up to the point in question (IIT Research Institute, 1980).

A computer program in which this representation is used is HULDEF, developed by the U.S. Navy for design use but now extended and made available to a number of shipyards in the U.S. for final hull form definition in the construction phase. HULDEF is said to be economical of computer time, and has been made compatible with other computer-based hull production programs. By HULDEF lines along the hull are developed from the given input waterlines and buttocks into *iso-girth* lines, formed by taking fixed percentages of the girthed length around each body plan station from centerline to deck edge (or chine) all along the hull from the tip of the bow to the stern. The lines are mathematized as parametric spline curves. The HULDEF system has been provided with interactive graphics capability so that the operator can readily display curves, first differences, and second differences, and can fair these on the scope to suit his own idea of fairness. This puts the fairing capability under the control and judgment of the operator just as it has been in the past under the control of the traditional drafter or mold loftsman. However, the previous time consuming operation of drawing the line—on a drafting table or on the mold loft floor—is no longer needed (Fuller, et al, 1977). Other similar systems are in use in some U.S. shipyards and abroad.

Computer applications in hydrostatic calculations are discussed in Section 5.16.

Section 2
Displacement and Weight Relationships

2.1 Archimedes' Principle. The fundamental physical law controlling the static behavior of a body wholly or partially immersed in a fluid is known as Archimedes' Principle which, as normally expressed, states that a body immersed in a fluid is buoyed up by a force that equals the weight of the displaced fluid.[2] Thus, the weight is considered to be a downward force that is proportional to the body's mass; the equal buoyant force is proportional to the mass of the displaced fluid.

Consider a body of fluid such as water, with a free surface, at rest. The fluid is of constant mass density, ρ (i.e., mass per unit volume). At any point P, a distance t below the free surface, the mass of fluid above the point is $\rho\ At$, where A is the cross sectional area parallel to the free surface of the column of fluid. In general, a fluid cannot support shear forces. Therefore, if the fluid be in a state of static equilibrium, it is necessary that equal forces be experienced in all directions at any such point. Since the gravitational force resulting from the mass of the fluid above is equal to its mass $\times g$, the pressure force experienced by the fluid at that point is $\rho g\ At$—or the weight of the column of fluid above P.

If a rigid body is afloat in the water in static equilibrium, Fig. 11, a consequence of the above reasoning is that the same pressure forces are directed normal to the surface of the body. The integration of the vertical component of all such pressures experienced by the surface S of the body is the buoyant force,

$$\sum_{o}^{S} \rho g t \cos \alpha\ \delta\ s,$$

where α is the inclination of any part of S from the horizontal. But $\sum_{o}^{S} t \cos \alpha\ \delta\ s$ represents the volume of the body beneath a plane coincident with the free surface. The weight of fluid would occupy this volume in the absence of the body is identically equal to ρg multiplied by the volume.

For the body to be in equilibrium, the integration of upward components of hydrostatic pressures over the surface of the body, or buoyancy, must be exactly balanced by the gravitational force of the body's mass, directed downward, i.e., its weight. Therefore, the weight of a ship and its contents is equal to the weight of displaced water, or displacement. Likewise, the mass of a ship and its contents is equal to the mass of displaced water. Hence, displacement can be expressed in either weight or mass units.

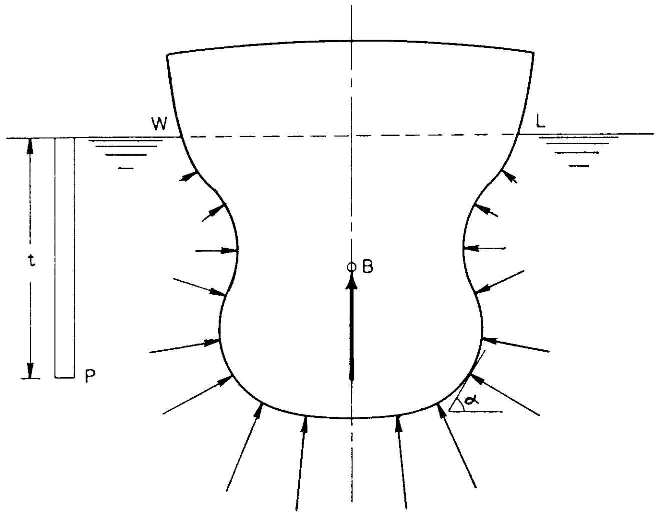


Fig. 11 Buoyant forces on a floating body

It is evident that a fully immersed rigid body, such as a submarine, also experiences an upward buoyant force equal and opposite to the weight of the water it displaces. A totally submerged body may weigh either more or less than the displaced water. For the body to be in equilibrium in its submerged position, it would have to receive, in the first case, an additional upward force, and in the second case, an additional downward force. When submerged and not resting on the bottom, a body may remain stationary, without rising or falling, only in the unusual case when its mass exactly equals the mass of the water it displaces.

2.2 Displacement and Center of Buoyancy. The volume of the underwater portion of a vessel may be calculated by methods outlined in Sections 4 and 5. The result is known as the volume of displacement, ∇, up to the waterline at which the vessel is floating.

If we know the mass density of the water, ρ, in which a ship is floating, we can calculate the weight of the displaced fluid, or the displacement weight, W,

$$W = \rho g \nabla. \tag{1}$$

By Archimedes' principle this weight is equal to the weight of the ship and its contents. In inch-pound (or "English") units, W is in long tons if ∇ is in ft^3 and $\rho g = 1/35.9$ long tons/ft^3 (62.4 lb/ft^3) in fresh water (FW) or $\rho g = 1/35.0$ long tons per ft^3 (64.0 lb/ft^3) in salt water (SW); i.e.,

$W = \nabla/35.9$ or $\nabla/35.0$ long tons (2240 lb per ton) in FW or SW, respectively.

[2] First stated about 250 BC by Archimedes, Greek mathematician and inventor (C.287-212BC).

In SI (*Système International*), the above expression for displacement weight (Eq. 1) applies if units of force are newtons (with ρ in kg/m^3) or kilonewtons (with ρ in t/m^3). In FW the value of ρg is approximately 9.81 kN/m^3 ($\rho = 1.0\ t/m^3$) and in SW ρg is 10.06 kN/m^3 ($\rho = 1.026\ t/m^3$). Such units are common in resistance and propulsion calculations (Chapter V).

However, adherance to the SI system obliges one to think of ship displacement, Δ, in mass units, rather than weight (force) units, with the unit of mass being a multiple of grams, such as a kilogram (1000 grams), or a metric ton (1000 kilograms) t, sometimes written as "tonne."[3] Hence, in the SI system, mass displacement,

$$\Delta = \rho \nabla, \tag{2}$$

where Δ is in metric tons, ∇ is in m^3, $\rho = 1.00\ t/m^3$ (equal to kg/L) in FW and $\rho = 1.026\ t/m^3$ in SW. For the above relationship to be true in inch-pound units, Δ would have to be expressed in lb-sec^2/ft or in the seldom-used *slugs* and ρ in lb-sec^2/ft^4 or in slugs/ft^3.

Since the mass density of fresh water is 1.0 kg/L or 1.0 t/m^3, density is numerically the same in SI units as specific gravity, γ (at standard temperature). Hence, it may be more convenient when using SI units to use,

$$\Delta = \gamma \nabla. \tag{3}$$

Again this is true in inch-pound units only if Δ is in lb-sec^2/ft or in slugs.

Sometimes naval architects prefer (as in Chapter II) to make use of the reciprocal of density, or specific volume, δ (volume per unit mass), in their calculations. For fresh water, of course, $\delta = 1.00\ m^3/t$; for salt water $\delta = 1/1.026 = 0.975\ m^3/t$.

We shall in general consider ship displacement in units of metric tons of mass, where one metric ton is equal to the mass of one cubic meter of fresh water (at standard temperature), i.e., $\rho = 1.0\ m^3/t$. It should be noted that one m^3 of fresh water in inch-pound units is 2204 lb or 0.9839 long ton (2240 lb/ton). Hence, it can be seen that one SI ton is roughly equivalent to a long ton of weight (1.6 percent error) in the inch-pound system. The term weight will often be used loosely to mean either weight in tons (lb) or mass in metric tons (kg.).

The centroid of the underwater portion of a vessel may be calculated by the principle of moments, using methods also outlined in Sections 4 and 5. The centroid is called the center of buoyancy. It represents a point through which the vertical buoyant vector is considered to pass, i.e., point B in Fig. 11.

2.3 Effect of Density of Medium. A decrease in the density of the fluid in which a vessel floats requires an increase in the volume of displacement ∇ in order to satisfy static equilibrium requirements. Therefore, a ship moving from salt water to fresh water, for example, experiences an increase in draft, δT. This increase can be calculated by equating the increase in displacement volume to the volume of a layer of buoyancy of uniform thickness, δT, distributed over the original load waterplane. The increase in displacement volume,

$$\nabla_F - \nabla_S = \nabla_S \frac{\rho_S}{\rho_F} - \nabla_S = \nabla_S \left(\frac{\rho_S}{\rho_F} - 1\right)$$

where subscript S refers to salt water, subscript F to fresh water. But, on the assumption that the ship is "wall-sided," the equal layer of buoyancy is,

$$\nabla_F - \nabla_S = A_{WP} \cdot \delta T$$

Hence,

$$A_{WP} \cdot \delta T = \nabla_S \left(\frac{\rho_S}{\rho_F} - 1\right)$$

and the increase in draft is,

$$\delta T = \frac{\nabla_S}{A_{WP}} \left(\frac{\rho_S}{\rho_F} - 1\right) = \frac{\nabla_S (\gamma_S - 1)}{A_{WP}} \tag{4}$$

where ∇ is displacement volume, ρ is mass density, A_{WP} is waterplane area, and $\frac{\rho_S}{\rho_F} = \gamma_S$ is specific gravity.

The centroid of the underwater body may shift, both vertically and longitudinally, with such a change in medium. In particular, an increase in draft as a result of a decrease in fluid density causes the vertical location of the center of buoyancy to rise with respect to the keel as a result of the increase in displacement volume, ∇.

When a ship becomes partially supported by mud of mass density ρ_M, the volume of displacement must decrease to the point where the sum of products of volume of displacement in the medium multiplied by the density of the medium equals the weight of the vessel. Correspondingly, the center of buoyancy may be found, using methods in Sects. 4 and 5 by calcu-

[3] Since this edition of *Principles of Naval Architecture* incorporates the transition from inch-pounds to SI units, reference will in general be made to both systems. The distinction, however, should always be borne in mind between inch-pound weight units and SI mass units.

lating the buoyant moment as the sum of products of buoyancy from each medium, multiplied by the distance to the centroid of each volume.

It is important to use the correct density of the water in making displacement calculations. There is about a $2\frac{1}{2}$ percent difference between the density of fresh water, as in the Great Lakes, and the salt water of the oceans. The water in some rivers and harbors and off the mouth of estuaries is usually brackish, and its density may vary considerably with the tides. When draft readings are taken to determine displacement, samples of the water should be taken at the same time in order to determine its density.

In principle, since the density of water changes slightly with temperature, a correction should be made to account for any differences from an agreed upon temperature standard. Furthermore, the temperature coefficient of expansion of steel may influence the volume of displacement of a steel vessel slightly up to any waterline if the temperature of the steel differs significantly from the standard.

2.4 Displacement vs. Weight Estimate. When preparing the design for a proposed ship, a careful estimate of its total weight and position of its center of gravity should be made, as discussed in Chapter II, Section 2. The total weight thus estimated may be compared later with the total displacement obtained from draft readings after the ship is afloat. If differences occur, as is usually the case, the error is assumed to be in the weight estimate. A check of the accuracy of the weight estimate for the vessel in its partially completed condition is usually first obtainable immediately after launching. As previously noted, these "weights" may be given in mass units.

Section 3
Coefficients of Form

3.1 General. In comparing ships' hull forms, displacements and dimensions, a number of coefficients are used in naval architecture. These coefficients are useful in power estimates and in expressing the fullness of a ship's overall form and those of the body plan sections and waterlines. Table 2 lists coefficients and particulars for a number of typical vessels, which will be found helpful in understanding the significance of the coefficients defined below.

Section 3 and Table 2 define and discuss the Block Coefficient, Midship Coefficient, Waterplane Coefficient Vertical Prismatic Coefficient, and Volumetric Coefficient. Table 2 also gives the general geometrical characteristics of 19 types of ships, ranging from a large, high-speed passenger liner capable of 33 knots sustained sea speed to a naval dock ship 171 m (555 ft) in length.

3.2 Definitions and Uses of Coefficients. *(a) Block Coefficient,* C_B. This is defined as the ratio of the volume of displacement ∇ of the molded form up to any waterline to the volume of a rectangular prism with length, breadth and depth equal to the length, breadth and mean draft of the ship, at that waterline.

Thus,

$$C_B = \frac{\nabla}{L \cdot B \cdot T}$$

where L is length, B is breadth and T is mean molded draft to the prevailing waterline. Practice varies regarding L and B. Some authorities take L as LBP, some as LWL, and some as an effective length, as discussed in Section 1.2. B may be taken as the molded breadth at the design waterline and at amidships, the maximum molded breadth at a selected waterline (not necessarily at amidships), or according to another standard. Most merchant ships have vertical sides amidships, with upper waterlines parallel to the centerline, thereby removing possible ambiguity in B.

Values of C_B at design displacement may vary from about 0.36 for a fine high-speed vessel to about 0.92 for a slow and full Great Lakes bulk carrier.

(b) Midship Coefficient, C_M. The midship section coefficient, C_M, sometimes called simply midship coefficient, at any draft is the ratio of the immersed area of the midship station to that of a rectangle of breadth equal to molded breadth and depth equal to the molded draft amidships.

Thus,

$$C_M = \frac{\text{Immersed area of midship section}}{B \cdot T}$$

Values of C_M may range from about 0.75 to 0.995 for normal ships, while for vessels of extreme form with a slack bilge and a hollow *garboard* area (immediately outboard of the keel) amidships, C_M might be as low as 0.62. In some cases vessels have been

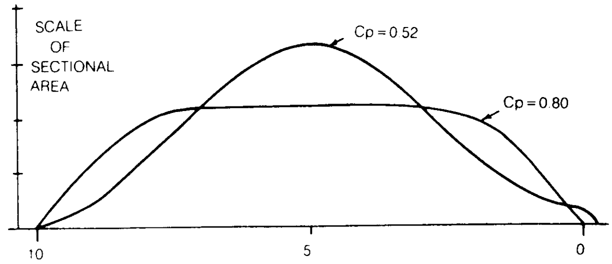


Fig. 12 Sectional area curves for different prismatic coefficients

built with bulges or *blisters* below the design waterline. Assuming B is taken at the prevailing waterline, then C_M may be greater than unity on such vessels.

(c) Prismatic Coefficient, C_P. The prismatic coefficient, sometimes called longitudinal prismatic coefficient, or simply longitudinal coefficient, gives the ratio between the volume of displacement ∇ and a prism whose length equals the length of the ship and whose cross section equals the midship section area.

Thus

$$C_P = \frac{\nabla}{L \times \text{immersed area of midship section}} = \frac{\nabla}{L \cdot B \cdot T \cdot C_M} = \frac{C_B}{C_M}.$$

The term longitudinal coefficient was originated and used by Adm. D. W. Taylor (1943) for the reason that this coefficient is a measure of the longitudinal distribution of a ship's buoyancy. If two ships with equal length and displacement have different prismatic coefficients, the one with the smaller value of C_P will have the larger midship sectional area ($B \cdot T \cdot C_M$) and hence a larger concentration of the volume of displacement amidships. This is clearly shown by Fig. 12, which compares the sectional area curves for two different vessels. The ship with the smaller C_P is also characterized by a protruding bulbous bow, which causes the swelling in the sectional area curve right at the bow, and its extension forward of Station *0*.

Prismatic coefficient is a frequently used parameter in studies of speed and power (Chapter V). Usual range of values is from about 0.50 to about 0.90. A vessel with a low value of C_P (or C_B) is said to have a fine hull form, while one with a high value of C_P has a full hull form.

(d) Waterplane Coefficient, C_{WP}. The waterplane coefficient is defined as the ratio between the area of the waterplane A_{WP} and the area of a circumscribing rectangle. Thus,

$$C_{WP} = \frac{A_{WP}}{L \cdot B}$$

As with the other coefficients, the length and breadth are not always taken in a standard way. The coefficient may be evaluated at any draft. The values of C_{WP} at the DWL range from about 0.65 to 0.95, depending upon type of ship, speed, and other factors.

(e) Vertical Prismatic Coefficient, C_{VP}. This coefficient is the ratio of the volume of a vessel's displacement to the volume of a cylindrical solid with a depth equal to the vessel's molded mean draft and with a uniform horizontal cross section equal to the area of the vessel's waterplane at that draft. This ratio is analogous to the prismatic or longitudinal coefficient, except that the draft and area of waterplane have been substituted for the vessel's length and area of midship section. The vertical prismatic coefficient of fineness is designated as C_{VP} and written as follows:

$$C_{VP} = \frac{\nabla}{C_{WP} \times L \times B \times T} = \frac{C_B}{C_{WP}}$$

(f) Volumetric Coefficient, C_V. This coefficient (or fatness ratio) is defined as the volume of displacement divided by the cube of one tenth of the vessel's length, or

$$C_V = \nabla/(L/10)^3$$

In essence, it is the dimensionless equivalent of displacement-length ratio, $\Delta \Big/ \left(\frac{L}{100}\right)^3$ frequently used in the past, where Δ is ship displacement in long tons in salt water, and L is ship length in feet. These coefficients express the displacement of a vessel in terms of its length. Ships with low volumetric coefficients might be said to be "thin", while those with a high coefficient are "fat." Values of the volumetric coefficient range from about 1.0 for light, long ships like destroyers, to 15 for short heavy ships like trawlers.

(g) Ratios of Dimensions. The three principal dimensions of the underwater body are sometimes referred to in ratio form. These are noted below, with approximate ranges for each:

Ratio of length to breadth	$= L/B$	Approx. range 3.5 to 10.
Ratio of length to draft	$= L/T$	Approx. range 10 to 30.
Ratio of breadth to draft	$= B/T$	Approx. range 1.8 to 5.

In view of the confusion which can arise when different definitions of dimensions—especially length—are used by different designers in forming the above coefficients and ratios, it is suggested that length between perpendiculars—on single-screw ships—and molded breadth at the design waterline and at amidships be used in forming these ratios. The length on the DWL is preferred for twin-screw ships (see Section 1.2). The definitions adopted should always be specified.

3.3 Geometrical Modification to Lines. It frequently happens during the design of a ship that unexpected requirements which were not foreseen necessitate a change in dimensions without changing the coefficients of form. Examples are an increase in breadth to provide greater stability, a decrease in design draft to allow entering a port with restricted water depth, or an increase in length to reduce wavemaking resistance. If this should happen after preliminary lines are faired, it seemingly requires that a

Table 2—Geometrical Characteristics of Typical Ships

	1 Pass. Liner	2 Cargo-Pass. Ship	3 Con-tainer Ship	4 Con-tainer Ship	5 Gen. Cargo Ship	6 Barge Carrier	7 Roll on/ Roll off Ship	8 Bulk Carrier	9 Gt. Lakes Ore Carrier
Length overall, m	301.75	166.60	262.13	185.93	171.80	272.29	208.48	272.03	304.80
Length between perpendiculars, L_{pp}, m	275.92	154.99	246.89	177.09	171.80	243.03	195.07	260.60	301.30
Length for coefficients, L, m	286.99	154.05	246.89	176.78	158.50	247.90	195.07	260.60	301.30
Molded depth to strength dk., m	22.63	14.66	20.12	16.61	13.56	18.29	21.18	19.05	14.94
Molded breadth, B, m	30.94	24.08	32.23	23.77	23.16	30.48	31.09	32.23	31.88
Molded draft for coeffs., T, m	9.65	8.23	10.67	8.23	8.23	8.53	9.75	13.96	7.85
Molded displacement, Δ, S.W., t	46,720	18,250	50,370	22,380	18,970	38,400	34,430	100,500	71,440
Block coefficient, C_B	0.532	0.583	0.579	0.630	0.612	0.582	0.568	0.836	0.924
Midship coefficient, C_M	0.953	0.967	0.965	0.975	0.981	0.922	0.972	0.996	0.999
Prismatic coefficient, C_P	0.558	0.603	0.600	0.646	0.624	0.631	0.584	0.839	0.924
Waterplane coefficient, C_W	0.687	0.725	0.748	0.740	0.724	0.765	0.671	0.898	0.975
Vertical prismatic coeff., C_{PV}	0.774	0.807	0.774	0.851	0.845	0.762	0.846	0.931	0.948
Longitudinal center of buoyancy from midship, % L	Amids.	Amids.	−1.1	−1.2	−1.5	−1.6	−2.4	+2.5	+0.5
Bulb area, % midship area	2.0	2.5	8.3	4.0	4.0	5.6	9.7	10.7	0
Volumetric coefficient, $(\nabla/L^3) \times 10^3$	1.93	4.87	3.26	3.95	4.65	2.46	5.18	5.54	2.55
L/B	9.28	6.40	7.94	7.44	6.84	8.13	6.27	8.09	9.45
B/T	3.21	2.93	2.91	2.89	2.81	3.57	3.19	2.31	4.06
Shaft horsepower, normal	158,000	18,000	43,200	19,250	17,500	32,060	37,000	24,000	14,000
Sea speed, knots	33	20	25	20	20	22	23	16.5	13.9
Froude number	0.320	0.265	0.261	0.427	0.261	0.229	0.270	0.168	0.132
Number of propellers, rudders	4,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,2

	10 Crude Oil Carrier	11 Petro-leum Prods. Tanker	12 LNG Tanker	13 Off-Shore Supply Vessel	14 Double-ended Ferry	15 Fishing Trawler	16 Arctic Ice-breaker	17 Naval Replen. Ship	18 Naval Frigate	19 Naval Dock Ship
Length overall, m	335.28	201.47	285.29	56.46	94.49	25.65	121.62	236.37	135.64	170.99
Length between perpendiculars, L_{PP}, m	323.09	192.02	273.41	53.19	91.59	23.04	106.98	234.70	124.36	164.59
Length for coefficients, L, m	323.09	192.02	273.41	53.19	91.59	23.75	107.29	234.70	124.36	164.59
Molded depth to strength dk., m	26.21	13.79	24.99	4.27	6.30	3.33	13.18	17.07	9.14	13.41
Molded breadth, B, m	54.25	27.43	43.74	12.19	19.81	6.71	23.77	32.72	13.74	24.99
Molded draft for coeffs., T, m	20.39	10.40	10.97	3.35	3.81	2.53	8.53	11.58	4.37	5.41
Molded displacement, Δ, S.W., t	308,700	43,400	97,200	1472.	2760.	222	10,900	52,140	3390	12,850
Block coefficient, C_B	0.842	0.772	0.722	0.660	0.392	0.538	0.488	0.569	0.449	0.563
Midship coefficient, C_M	0.996	0.986	0.995	0.906	0.732	0.833	0.853	0.987	0.741	0.933
Prismatic coefficient, C_P	0.845	0.784	0.726	0.729	0.534	0.646	0.572	0.577	0.605	0.603
Waterplane coefficient, C_W	0.916	0.854	0.797	0.892	0.702	0.872	0.740	0.734	0.727	0.720
Vertical prismatic coeff., C_{PV}	0.919	0.904	0.906	0.740	0.558	0.617	0.660	0.779	0.618	0.782
Longitudinal center of buoyancy from midship, % L	+2.7	+1.9	Amids.	−0.3	Amids.	−1.7	+1.3	−0.9	−1.4	−1.4
Bulb area, % midship area	0	0	9.7	0	0	0	0	10.0	0	2.0
Volumetric coefficient, $\nabla/L^3) \times 10^3$	8.9	5.98	4.64	9.53	3.51	16.2	8.97	3.9	1.7	2.8
L/B	5.96	7.00	6.25	4.35	4.62	3.54	4.51	7.17	9.05	6.59
B/T	2.66	2.64	3.99	3.33	5.20	2.65	2.79	2.82	3.14	4.62
Shaft horsepower, normal	35,000	15,000	34,400	3,740	7,000	500	18,000	100,000	40,000	22,900
Sea speed, knots	15.2	16.5	20.4	12	16.1	10.7	18	26	30	21.5
Froude number	0.139	0.196	0.203	0.270	0.276	0.361	0.285	0.279	0.442	0.275
Number of propellers, rudders	1,1	1,1	1,1	2,2	2,0	1,1	3,1	2,2	1,1	2,2

1) Vessel 10 has a cylindrical bow. 2) Vessel 14 has vertical axis propellers and a fixed skeg at each end.

completely new set of lines be drawn and faired. However, by making systematic changes in the offsets, it may be possible to accomplish the desired transformation without disturbing the fairness of the lines, and without necessitating complete recalculation of the curves of form.

For example, a simple respacing of body plan stations leads to an elongation or shortening of the lines at constant breadth and draft, with displacement changing in direct proportion to the station spacing; the form coefficients C_B, C_P, C_M, C_{WP} and C_{VP} will not change, and the fairness of lines will be preserved. Of the curves of form, changes will be experienced only in those quantities which depend upon length and displacement, including C_V. Correspondingly, an increase in waterline spacing leads to a proportionate change in displacement with no change in C_B, C_P, C_M, C_{WP} and C_{VP}. Those curves of form which are dependent upon displacement and draft are the only ones which will change. Similar conclusions are reached insofar as changes in buttock spacing—that is, changes in halfbreadth—are concerned.

The combined effect of two or more of such changes is multiplicative. For example, if the length of the vessel were to increase 10 percent by an increase in station spacing, the breadth were to increase 5 percent by an increase in halfbreadths, and the draft were to decrease 8 percent by a reduction in waterline spacing, the resulting volume of displacement ∇_2 would be obtained from ∇_1, the initial volume of displacement, by,

$$\nabla_2 = 1.1 \cdot 1.05 \cdot 0.92 \cdot \nabla_1 = 1.0626\ \nabla_1$$

A new body plan, waterlines plan and profile could be drawn directly, in which new longitudinal distances x_2 are obtained from old longitudinal distances x_1 by $x_2 = 1.1\,x_1$; new halfbreadths y_2 are obtained from old halfbreadths y_1 by $y_2 = 1.05\,y_1$; etc.

Changes in the more important curves of form, defined in Section 5, would give,

$$\overline{KB}_2 = 0.92\ \overline{KB}_1$$

$$TP\,cm_2 = 1.1 \cdot 1.05 \cdot TP\,cm_1$$

$$LCF_2 = 1.1\ LCF_1$$

$$LCB_2 = 1.1\ LCB_1$$

$$\overline{KM}_2 = \overline{KB}_2 + \overline{BM}_2$$

$$= 0.92\ \overline{KB}_1 + \left(\frac{1.1 \cdot (1.05)^3}{1.1 \cdot 1.05 \cdot 0.92}\right) \overline{BM}_1$$

$$\overline{KM}_{L_2} = 0.92\ \overline{KB}_1 + \left(\frac{(1.1)^3 \cdot 1.05}{1.1 \cdot 1.05 \cdot 0.92}\right) \overline{BM}_{L_1}$$

Wetted surface, which depends upon girthed distances, does not vary in a simple manner and would have to be recomputed for the transformed design.

Methods have been developed (Rawson & Tupper, 1983) to estimate modifications to the geometrical quantities on the basis of partial derivatives. Inasmuch as these methods assume infinitesimal changes in the independent variables, L, B, etc., they may lead to inaccuracies in practical use. On the other hand, direct calculations to find the transformed quantities are by their nature both exact and correct, and therefore they are recommended.

A traditional and practical way of shifting the LCB of a new design without changing displacement is known as the method of *swinging stations.* Fig. 13 shows the sectional area curve of a ship and the centroid of the area under the curve, the latter having been found from both axes ($\bar{x}$ and $\bar{y}$). If the centroid now be moved forward (or aft) a distance $\delta\bar{x}$, and a straight line be drawn through the shifted position and original base, it will establish an angle γ by which all points on the curve may be similarly shifted so that the desired shift of LCB occurs. Any original body plan station such as station 3 must then be shifted by distance δx. This allows one to find the shift of any offset (height or halfbreadth) forward or aft directly from the transformed sectional area curve. Hence, the waterlines and profile views on the lines plan may be redrawn without refairing being required. From the redrawn waterlines and profile a new body plan, with equally spaced stations, may then be constructed.

A somewhat similar transformation can be done to the separate ends of a sectional area curve with some parallel middle body if one wishes to change the fullness of the design. Let us suppose the forebody of a given sectional area curve has a prismatic coefficient of C_{PF1}, but it is desired to increase this by respacing stations to gain more displacement. The new forebody prismatic is to be C_{PF2}. Thus $\delta C_{PF} = C_{PF2} - C_{PF1}$. Then it can be shown (Lackenby, 1950) that if x_1 is the dimensionless distance from the left-hand axis of the curve, where x_1 lies between 0 and 1.0, the shift forward δx to give the required new prismatic coefficient of the forebody is obtained from,

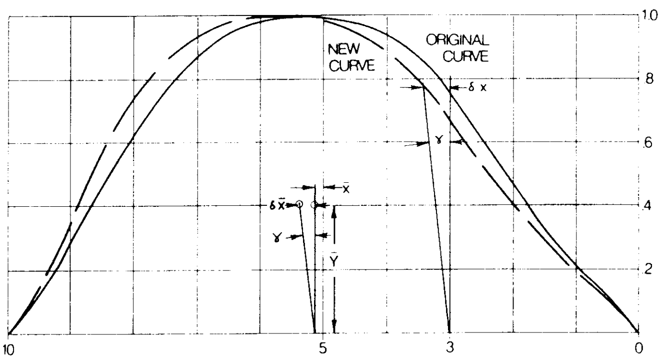


Fig. 13 Swinging stations method of modifying sectional area curve

$$\delta x/(1 - x_1) = \delta C_{PF}/(1 - C_{PF1}),$$

or

$$\delta x = \delta C_{PF}[(1 - x_1)/(1 - C_{PF1})]. \quad (5)$$

This procedure, which is known as the *one-minus-prismatic* rule, is illustrated in Fig. 14 (Lackenby, 1950). Having modified the sectional area curve in the indicated way, body plan stations must now be shifted the indicated amount. Thus, the waterlines and profile views in the entrance may be redrawn, with a new body plan for the forebody to suit equally spaced stations. It should be noted, however, that having first transformed the forebody a similar transformation of the afterbody in general leads to a combined longitudinal center of buoyancy of the entire ship which will differ from that of the basic ship before the transformation.

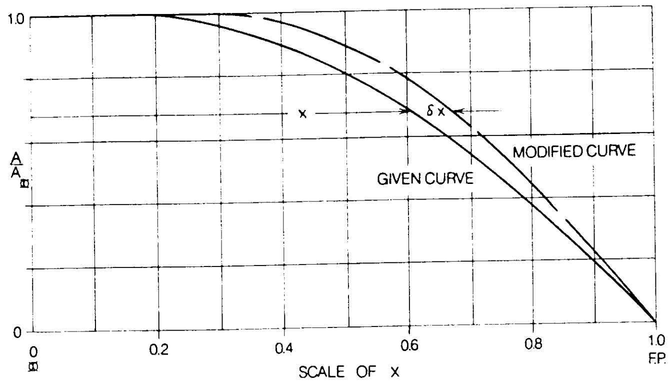


Fig. 14 One-Minus-Prismatic method of modifying sectional area curve

Söding, et al (1977) show an extensive transformation of an existing containership design to a design of widely different particulars following generally the methods of Lackenby (1950).

Section 4
Integrating Rules and Methods

4.1 General. For a variety of reasons, it is necessary to be able to calculate areas, centroids, volumes—and other geometrical characteristics—of a ship's form when floating at any prescribed waterline. Areas of the immersed cross sectional area at each body plan station and of each waterplane are of particular interest, not only for their own sake but because—as will be shown later—volumes can be calculated from areas. Because of the symmetry of the two sides of most vessels, most of these calculations need be performed for only one side of the ship and then multiplied by 2.

Each of the half transverse sections, or half waterplanes, form a closed curve, such as *OABD—GH* in Fig. 15. The area enclosed may be found by integral calculus, provided *AB—G* is a curve whose mathematical equation is known. Inasmuch as most ship curves are not mathematical curves, it is customary to approximate the area by numerical integration.

An important property of such a closed curve is its centroid, which is located at a distance from the axis *OY* equal to $\bar{x}$, where $\bar{x}$ is the quotient of the first moment of the area about axis OY divided by the area itself. If the curve *OABD—GH* were to represent a thin lamina of uniform density and of constant thickness, then the centroid would represent the location of its center of mass (generally known as center of gravity).

4.2 Formulas For Area, Moment, Centroid, Moment of Intertia, and Gyradius. In Fig. 15 the area enclosed by the x and y-axes and the curve *ABDG* may be considered as comprised of many small rectangles such as *NBPQ*, of dimensions y and δx, where δx is very small. Using methods of the calculus, we may derive expressions for the area of the curvilinear figure and for various properties of the area.

(a) Areas. Let δA be the area of the elementary rectangle *NBPQ*. Then $\delta A = y\delta x$, and the entire area under the curve, A, is given by the summation of all such elementary areas, or,

$$A = \Sigma \delta A = \Sigma y \delta x.$$

Putting this in the form of a definite integral between the limits 0 and H,

$$A = \int_0^H y dx. \tag{6}$$

(b) Moments and *Centroids.* Let δM_ℓ be the first moment of the area of the elementary rectangle *NBPQ* about axis *OY*. Then $\delta M_\ell = (\delta A)x = xy\delta x$. Hence, the moment of the entire area under the curve about axis *OY* may be written as $M_\ell = \Sigma xy\delta x$, which may be expressed as the definite integral,

$$M_\ell = \int_0^H xydx. \tag{7}$$

The distance $\bar{x}$ of the centroid of the area from axis *OY* is given by the quotient of moment about *OY* divided by area or,

$$\bar{x} = \frac{\int_0^H xydx}{\int_0^H ydx}. \tag{8}$$

Let δM_t be the first moment of the elementary area $NBPQ$ about the baseline OX. Then

$$\delta M_t = (\delta A)\frac{y}{2} = \frac{y^2}{2}\,\delta x.$$

The moment of the entire area about the baseline becomes,

$M_t = \frac{1}{2}\Sigma y^2 \delta x$, or in the form of an integral,

$$M_t = \frac{1}{2}\int_0^H y^2 dx. \qquad (9)$$

The distance $\bar{y}$ of the centroid of the area from the baseline OX is the quotient of moment about the baseline divided by area, or,

$$\bar{y} = \frac{\frac{1}{2}\int_0^H y^2 dx}{\int_0^H y dx}. \qquad (10)$$

c Moments of Inertia and Gyradii. Let δI_ℓ be the second moment, or moment of inertia, of the area of the elementary rectangle $NBPQ$ about axis OY. Then $\delta I_\ell = (\delta A)\, x^2 = x^2 y \delta x$. Hence the moment of inertia of the entire area under the curve about OY, I_ℓ, is,

$$I_\ell = \sum x^2 y \delta x \text{ or } I_\ell = \int_0^H x^2 y dx. \qquad (11)$$

The gyradius r_ℓ of the area about axis OY is given by the square root of the quotient of moment of inertia divided by area, or,

$$r_\ell = \sqrt{\frac{\int_0^H x^2 y dx}{\int_0^H y dx}}. \qquad (12)$$

If $I_{g\ell}$ be the longitudinal moment of inertia of the area under the curve about a transverse axis through the centroid (axis parallel to the Y-axis), we have by the parallel axis principle of mechanics, $I_{g\ell} = I_\ell - A\bar{x}^2$.

The area under the curve AG may also be considered as comprised of many small squares such as $\delta x \delta y$, Fig. 15. Then let δI_t be the second moment, or moment of inertia, of the area of the elementary square about the baseline OX. But $\delta I_t = \delta x \delta y \cdot y^2$. Thus the moment of inertia of the entire area under the curve about the baseline I_t may be written as $I_t = \sum\sum \delta x \delta y \cdot y^2$, or,

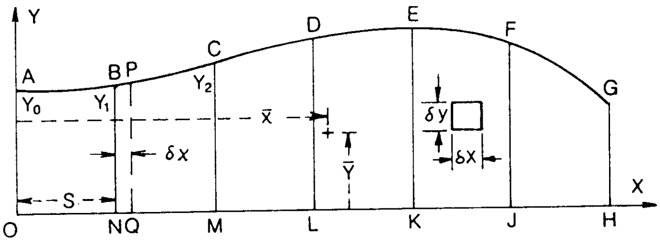


Fig. 15 Curve to be integrated

$$I_t = \int_0^H \int_0^y y^2\, dy\, dx.$$

Since

$$\int_0^y y^2 dy = \frac{1}{3}y^3, \text{ then } I_t = \frac{1}{3}\int_0^H y^3 dx. \qquad (13)$$

The gyradius r_t of the area about the baseline OX is given by,

$$r_t = \sqrt{\frac{\frac{1}{3}\int_0^H y^3 dx}{\int_0^H y dx}}. \qquad (14)$$

In order to evaluate these integrals, naval architects again overcome the limitation that most ship lines are not represented by mathematical formulas by utilizing approximate rules of integration. A rule of integration assumes that the curve to be integrated is closely approximated by a mathematical curve that has the same offsets (or ordinates) as the actual ship curve at a series of stations. The desired integrals are then approximated by taking the sum of products of offsets and particular multipliers developed for each rule and multiplying the sum by an integrating factor, as described in the following subsections.

4.3 Trapezoidal Rule. In Fig. 15 each portion of the curve $AB—G$ between pairs of ordinates as AB, BC, etc. is considered to be approximated by a straight line through each pair of points. If the spacing between each pair of ordinates is s, then the area of trapezoid $OABN = s \cdot \frac{1}{2}(y_0 + y_1)$, the area of trapezoid $NBCM = s \cdot \frac{1}{2}(y_1 + y_2)$ and the area of trapezoid $JFGH = s \cdot \frac{1}{2}(y_{n-1} + y_n)$. If the areas of all n trapezoids are added, their combined area, and the approximate area A under the curve is,

$$A = s\,(\tfrac{1}{2}y_0 + y_1 + y_2 + \ldots + y_{(n-1)} + \tfrac{1}{2}y_n) \qquad (15)$$

This is known as the *trapezoidal rule* for area.

Rules for moments of the area based upon the trapezoidal rule may be derived. Thus, the moment M_ℓ of the combined area of all the trapezoids about axis OY is,

$$M_\ell = s^2\left[\frac{1}{6}y_0 + y_1 + 2y_2 + \dots + (n-1)y_{(n-1)} + \left(\frac{3n-1}{6}\right)y_n\right]. \quad (16)$$

Correspondingly, the moment of inertia I_ℓ of the combined area of all the trapezoids about the OY axis is,

$$I_\ell = s^3\left[\frac{1}{12}y_0 + \frac{7}{6}y_1 + \frac{25}{6}y_2 + \dots + \left(\frac{6n^2-12n+7}{6}\right)y_{(n-1)} + \left(\frac{6n^2-4n+1}{12}\right)y_n\right]. \quad (17)$$

The trapezoidal rule may be adapted to give transverse moment M_t and transverse moment of inertia I_t, but the expressions are complicated by the presence of products of the ordinates y_0y_1, y_1y_2, etc. for M_t and $y_0^2y_1$, $y_0y_1^2$, $y_1^2y_2$, $y_1y_2^2$, etc. for I_t. To overcome this complexity the squares and cubes of the ordinates as given by the integrals in Sect. 4.2 are sometimes weighted by the trapezoidal area rule multipliers to give rough approximations of moment and moment of inertia about the x-axis.

Owing to the straight line approximation inherent in the trapezoidal rule, a closer spacing of ordinates is needed to approach the same level of accuracy for area obtainable with other rules described later, and its application is limited in naval architectural calculations to finding areas. In the case of a convex curve with no point of inflection, the area found by the trapezoidal rule is always less than the true area.

4.4 Simpson's First Rule. This rule, and that to follow in 4.5, are part of a group of rules known as Newton-Cotes Rules. *Simpson's First Rule* rigorously integrates the area under a curve of the type $y = a + bx + cx^2$, which is a second order parabola, or polynomial of degree 2, by applying multipliers to groups of three equally spaced ordinates. That is, if the portion of the curve in Fig. 15 extending from A to C is parabolic, and the ordinates y_0, y_1, and y_2 are equally spaced, then the area found by Simpson's First Rule is precisely correct. Inasmuch as many ship curves are not dissimilar to the parabola, the area so found is a close approximation to that of the ship, and the rule is widely used in naval architecture.

The rule may be derived by assuming the area is given by the expression, $A = k_0y_0 + k_1y_1 + k_2y_2$. Given the mathematical form of the curve, ($y = a + bx + cx^2$) and ordinates at spacing s, then $y_0 = a$, $y_1 = a + bs + cs^2$ and $y_2 = a + 2bs + 4cs^2$. Putting the three y values into the expression for A, an equation for the coefficients a, b and c results. But A is also equal to the definite integral,

$$A = \int_0^{2s} y\,dx = \int_0^{2s} (a + bx + cx^2)\,dx = 2as + 2bs^2 + \frac{8}{3}cs^3.$$

Equating the two expressions for A, we may set the coefficients of a, b, and c equal to each other. There are three resulting equations in the three unknowns, k_0 k_1 and k_2, which may be solved simultaneously. This gives,

$$k_0 = \frac{s}{3},\ k_1 = \frac{4s}{3},\ k_2 = \frac{s}{3}.$$

The curve to be integrated must be divided into an even number of spaces by equally spaced ordinates. The multipliers for even numbered ordinates are then found on the assumption that each such ordinate represents the termination of one parabolic curve and the initiation of another. Knuckles in the curve are allowed at these ordinates. Hence the multiplier for such even numbered ordinates (except for the first and last) is 2, giving the following form of the rule,

$$A = \frac{s}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 \dots + 4y_{(n-1)} + y_n] \quad (18)$$

where n is even. In order to simplify the multipliers when using the First Rule, it is not uncommon to divide them by 2, in which case they are known as *half multipliers.* The final integration is then found by multiplying the summed products of ordinates and multipliers by an additional factor of 2, so that the integrating factor becomes $2s/3$.

Simpson's First Rule may be adapted to the calculation of longitudinal moment M_ℓ and longitudinal moment of inertia I_ℓ in a similar way to that used for finding a formula for area, with the assumption that the ordinates of a 2nd order parabolic curve are xy, and x^2y, respectively.

In practice, it is customary to perform calculations for area, longitudinal moment, and longitudinal moment of inertia using Simpson's First Rule by means of tables such as Table 6, described in sections 5.3, 5.4, and 5.5. Separate columns are provided in the table for the ordinates, for Simpson's Multipliers, for levers (for longitudinal moment), for the squares of levers (for longitudinal moment of inertia), and for the products of the ordinates times the levers times Simpson's Multipliers, etc. For simplicity, the levers are usually non-dimensionalized by dividing by the station spacing, s. When this is done, the tabular calculations for M_ℓ and I_ℓ (axis of moments at origin, often located amidships) may also be expressed by the following formulas, which may be found more appropriate for computer programming, taking the axis at $x = 0$,

$$M_\ell = \frac{s^2}{3}[4y_1 + 4y_2 + 12y_3 + \ldots + 2(n-2)y_{(n-2)} + 4(n-1)y_{(n-1)} + ny_n] \quad \text{where } n \text{ is even.} \tag{19}$$

$$I_\ell = \frac{s^3}{3}[4y_1 + 8y_2 + 36y_3 + \ldots + 2(n-2)^2y_{(n-2)} + 4(n-1)^2y_{(n-1)} + n^2y_n] \quad \text{where } n \text{ is even.} \tag{20}$$

It will be noted that there are no y_0 terms above because the axis for moments is at $x = 0$ where $y = y_0$ and the lever arm is zero.

If similar derivations are applied to the determination of formulas for transverse moment of area M_t and transverse moment of inertia of area I_t it will be found that expressions of the form,

$$M_t = k_0 y_0^2 + k_1 y_1^2 + k_2 y_2^2 \text{ and}$$
$$I_t = k_0 y_0^3 + k_1 y_1^3 + k_2 y_2^3,$$

cannot be solved, owing to an excess of equations. This results from the presence of cross products of the ordinates, as noted in Sect. 4.3. Nevertheless, Simpson's First Rule is routinely applied to the calculation of transverse moment of area, and transverse moment of inertia of area, by weighting the squares and cubes of ordinates by Simpson's area multipliers, and in accordance with the integrals in Section 4.2. This is equivalent to assuming that the ordinates of the 2nd order parabola are y^2, and y^3, respectively.

Therefore, in the event the squares of the ordinates of the curve to be integrated, or the cubes of the ordinates, respectively, followed a 2nd order parabolic curve, the integration for transverse moment, and for transverse moment of inertia, by Simpson's First Rule, would be precisely correct.

Table 6 includes columns for the cubes of ordinates and for the products of these times Simpson's Multipliers, in order to calculate transverse moment of inertia (about the ship's centerline) by Simpson's First Rule.

It may be shown that Simpson's First Rule also precisely integrates the area under a third order parabolic curve of the form,

$$y = a + bx + cx^2 + dx^3,$$

which passes through the three given ordinates. Hence, Simpson's First Rule is accurate enough for most ship problems.

4.5 Simpson's Second Rule. This rule correctly integrates the area under a third order parabolic curve, or polynomial of degree 3, when four equally spaced ordinates are provided. The derivation of appropriate Simpson's multipliers is achieved using similar steps to those outlined in Sect. 4.4. It may be shown that if we assume $A = k_0 y_0 + k_1 y_1 + k_2 y_2 + k_3 y_3$, then, $k_0 = k_3 = 3s/8$, and $k_1 = k_2 = 9s/8$ where s is the station spacing. Thus, in general, the area A under an arbitrary curve by *Simpson's Second Rule* is

$$A = \frac{3s}{8}[y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + \ldots\ldots + \ldots\ldots + 2y_{(n-3)} + 3y_{(n-2)} + 3y_{(n-1)} + y_n], \; (n = 3, 6, 9 \ldots \text{etc.}) \tag{21}$$

As with Simpson's First Rule, a separate parabolic curve is assumed between the extremities of each group of intervals—three intervals in the case of the Second Rule—and knuckles in the curve to be integrated are permitted at these points.

Simpson's Second Rule may be applied to the calculation of longitudinal moment of area M_ℓ and longitudinal moment of inertia of area I_ℓ by combining $3s^2/8$ and $3s^3/8$, respectively, with the Simpson's multipliers for area, together with non-dimensionalized levers and levers squared, respectively, as done when using the First Rule. The resulting M_ℓ and I_ℓ are not rigorously correct for a parabolic curve of the third order, but are routinely used. The resulting errors are quite small, in general.

The accuracy of transverse moment of area M_t and transverse moment of inertia of area I_t when calculated by Simpson's Second Rule is subject to the same limitations as apply to the First Rule. However, the rule is routinely used for these purposes.

4.6 Single Interval Rules. These rules allow one to find area under the curve, A, longitudinal moment of area M_ℓ, and longitudinal moment of inertia of area I_ℓ about axis OY for a single interval between the first two ordinates of a second order parabola of the form $y = a + bx + cx^2$ when the curve is defined by three equally-spaced ordinates with spacing s.

Consider Fig. 15. The *five, eight, minus one* rule states that the area A between ordinates y_0 and y_1 is,

$$A = \frac{s}{12}(5y_0 + 8y_1 - y_2). \tag{22}$$

The *three, ten, minus one* rule states that the longitudinal moment M_l of the area between y_0 and y_1 about axis OY is,

Table 3—Newton-Cotes Rules
Ordinates equally spaced, with end ordinates at ends of curve

Number of Ordinates	Multipliers for ordinate numbers 1	2	3	4	5	6	7	8	9
2	1/2	1/2							
3	1/6	4/6	1/6						
4	1/8	3/8	3/8	1/8					
5	7/90	32/90	12/90	32/90	7/90				
6	19/288= 0.0660	75/288= 0.2604	50/288= 0.1736	50/288= 0.1736	75/288= 0.2604	19/288= 0.0660			
7	0.0488	0.2571	0.0321	0.3238	0.0321	0.2571	0.0488		
8	0.0435	0.2070	0.0766	0.1730	0.1730	0.0766	0.2070	0.0435	
9	0.0349	0.2077	−0.0327	0.3702	−0.1601	0.3702	−0.0327	0.2077	0.0349

Area = Σ (Multipliers × Ordinates) × distance between end ordinates, R

$$M_\ell = \frac{s^2}{24}(3y_0 + 10y_1 - y_2). \quad (23)$$

A similar rule may be derived for longitudinal moment of inertia I_ℓ of the area between y_0 and y_1 about axis OY. It might be called the *seven, thirty six, minus three* rule and is,

$$I_\ell = \frac{s^3}{120}(7y_0 + 36y_1 - 3y_2). \quad (24)$$

These rules are exact for the 2nd order parabolic curve assumed.

4.7 Higher Order Curves. In the event a curve is believed to be more closely approximated by a higher order parabola, or polynomial of higher degree, the Newton-Cotes multipliers may be used, but a greater number of equally spaced ordinates is needed in way of that portion of the curve over which the defining parabola is assumed to hold.

Thus, five equally spaced ordinates are needed to define a curve in the form $y = a + bx + cx^2 + dx^3 + ex^4$. It may be shown that the area A under such a curve is given by,

$$A = \frac{s}{45}[14y_0 + 64y_1 + 24y_2 + 64y_3 + 14y_4]. \quad (25)$$

By combining end ordinates for two or more groups of four equal intervals, a rule analagous to Simpson's First or Second Rule may be devised.

Based upon Miller (1963), multipliers for higher order curves would be as shown in Table 3. In each case, the area under the curve would be the product of the distance between end ordinates R and the sum of products of multipliers and ordinates. It may be noted that the sum of the ordinates in Table 3 equals 1.0 for each polynomial.

4.8 Half-Spaced Ordinates. Near the ends of a ship it is customary to introduce additional body plan stations midway between pairs of the normal 10 or 20 stations in the length between perpendiculars. This is done to better define the hull form in these regions, as it is usually changing more rapidly with longitudinal distance than near midship. In order to improve the accuracy of integration, one may take advantage of offsets at such half-spaced stations—i.e., stations ½, 1½, 18½ and 19½ in a 20-station length. The foregoing rules of integration may be easily modified to this end. The modification implies that the distance over which the curve to be integrated may be assumed to match the hypothetical curve is cut in half.

In the above case, assuming Simpson's First Rule, each separate parabolic curve would be assumed to extend from station 0 to 1, 1 to 2, 18 to 19 and 19 to 20, while in the middle portion of the ship separate parabolas would be considered to extend from station 2 to 4, 4 to 6 . . . 16 to 18. In order to accommodate this combination of spacings, the Simpson's multipliers are reduced to one half their normal values in way of the half stations.

Fig. 16 shows the arrangement of half stations at the end of a 10-station ship, along with the Simpson's half multipliers appropriate to the First Rule.

An important consideration in any ship calculation is whether the area (or quantity being integrated) is complete, or applies to one side of the ship only. If the latter, a factor of 2 must be introduced into the calculation to obtain the total for the ship.

4.9 Tchebycheff's Rules. The Tchebycheff Rules use varying numbers of ordinates located at irregular intervals along the base line, spaced in such a way that the sum of the ordinates is directly proportional to the area under the curve. The curve to be integrated is assumed parabolic, i.e., $y = a + bx + cx^2 \ldots + kx^n$. The number of ordinates needed is the same as the order of parabola assumed. The length of curve is taken as $2s$, as in Fig. 17. The validity of the rule is based upon the location of the ordinates, which are symmetrically disposed about the middle, such as y_1 and y_2 at locations x and $-x$.

Consider a second order parabola, with origin at *0*, as shown in Fig. 17. Assume the area under the curve from D to E is given by,

$$A = p(y_1 + y_2),$$

$$y_1 = a - bx + cx^2, \qquad y_2 = a + bx + cx^2$$

$$\therefore y_1 + y_2 = 2a + 2cx^2$$

Hence, $A = 2pa + 2pcx^2$.

$$\text{But } A = \int_{-s}^{s} ydx = \left[ax + b\frac{x^2}{2} + c\frac{x^3}{3}\right]_{-s}^{s}$$

$$= 2as + 2c\left(\frac{s^3}{3}\right).$$

Equating coefficients of a and c,

$$2p = 2s \therefore p = s;$$

$$2px^2 = 2\left(\frac{s^3}{3}\right) \therefore x^2 = \frac{s^3}{3p} = \frac{s^2}{3} \text{ and}$$

$$x = \frac{s}{\sqrt{3}} = 0.57735s.$$

Then the area,

$$A = s(y_1 + y_2), \qquad (26)$$

where y_1 and y_2 are at locations $\pm$ 0.57735s from the origin.

Table 4 shows the locations of ordinates for numbers of ordinates up to 10. It will be noted that in the case of an odd number the middle ordinate is at the middle of the curve, or origin of Fig. 17. In each case, the area under the curve is found as the average length of ordinate $\frac{1}{n}(y_1 + y_2 \ldots + y_n)$ multiplied by the length of the base line 2s.

4.10 Integration For Arbitrarily Spaced Ordinates. It frequently happens that when integrating a ship's waterplane, the extremities of the curve, either at bow or stern, do not fall at integral stations—in particular the FP or AP—but instead the curve originates either forward of or abaft the perpendicular.

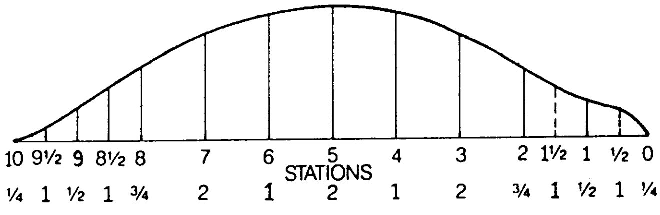


Fig. 16 Half-spaced ordinates

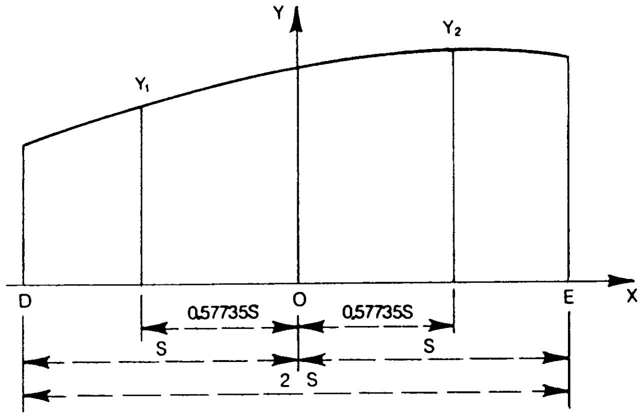


Fig. 17 Integration by Tchebycheff Rule

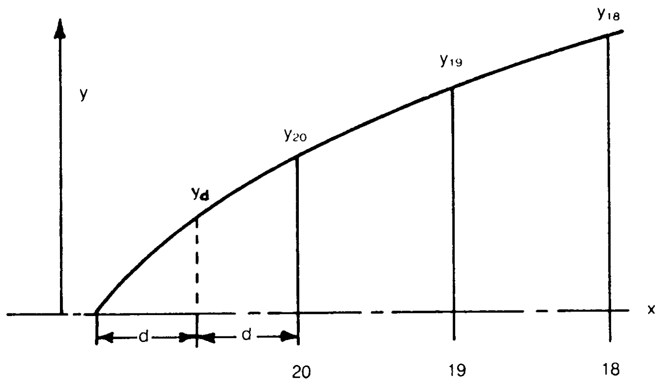


Fig. 18 Curve ending beyond perpendicular

Table 4—Spacing of Tchebycheff's Ordinates

Number of ordinates used	Positions of ordinates from middle of base, in fractions of half-length (s) of base
2	0.5773
3	0 0.7071
4	0.1876 0.7947
5	0 0.3745 0.8325
6	0.2666 0.4225 0.8662
7	0 0.3239 0.5297 0.8839
8	0.1026 0.4062 0.5938 0.8974
9	0 0.1679 0.5288 0.6010 0.9116
10	0.0838 0.3127 0.5000 0.6873 0.9162

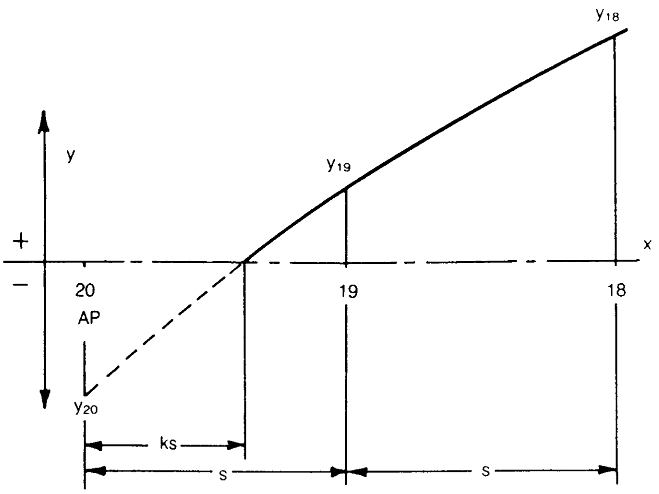


Fig. 19 Curve ending before perpendicular

Fig. 18 shows such a curve extending abaft the AP, or station 20. To handle this situation, one may add and measure an additional ordinate midway between AP and 0-ordinate and add a Simpson's First Rule integration to that for the waterline as a whole, using methods in Sect. 4.11.

In case the curve terminates forward of AP, as in Fig. 19, the curve may be extended by the dotted line and a fictitious negative ordinate, or ordinates, such as y_{20}, may be read. Assuming the extremity of the curve is at ks, where s is the station interval, and the curve is a second order parabola, it may be shown that the area A under the real part of the curve abaft station 18 is given by,

$$A = \frac{s}{3}\left[\left(1 - 3k + \frac{9}{4}k^2 - \frac{1}{2}k^3\right)y_{20} + \left(4 - 3k^2 + k^3\right)y_{19} + \left(1 + \frac{3}{4}k^2 - \frac{1}{2}k^3\right)y_{18}\right] \quad (27)$$

This form might be known as the *partial area rule.* Depending upon whether k is less or more than unity, either y_{20} or y_{20} and y_{19} would be negative.

As an example, in Fig. 19, let $k = 0.3$. Then the calculation for area under the curve A from its extremity to station 18 would be,

$$A = \frac{s}{3}[0.289y_{20} + 3.757y_{19} + 1.054y_{18}]. \quad (28)$$

It may be seen that this formula reverts to Simpson's First Rule when $k = 0$. The *five, eight, minus one rule* multipliers result when $k = 1.0$.

Additional integrating rules could be derived using non-equally-spaced stations, with multipliers dependent upon the specific station locations chosen and predicated on the use of a parabolic curve, but there has in the past been little need for these.

4.11 Combining Rules for Any Number of Ordinates. Ship curves are sometimes divided by a number of equally spaced ordinates which are incompatible with the number needed for integration by either Simpson's First or Second Rules—for example, in the case of 5, 7, 9 or 11 intervals, defined by 6, 8, 10 and 12 ordinates. However, these cases are readily handled by a combination of the two rules. In case both rules are used to integrate such a curve, one may perform a separate integration for each portion of the curve, or a single integration for the entire curve, in which case Simpson's multipliers for the "secondary" portion are modified to suit the integrating factor appropriate to the "primary" portion.

For example, assume there are eight ordinates and the area is to be found using the First Rule over the primary portion. Second Rule multipliers must then be multiplied by the factor $\frac{3s}{8} \div \frac{s}{3} = \frac{9}{8}$. The calculation takes the form shown by Table 5. The integrating factor is $\frac{s}{3}$, appropriate to the First Rule. Also shown in Table 5 are multipliers when the primary portion is integrated using the Second Rule.

4.12 Polar Integration. Whereas most ship curves are defined in rectangular coordinates, there are cases where polar coordinates are more convenient. For example, in calculations relating to static stability, portions of transverse sections of a ship might be wedge shaped such as $OACO$ in Fig. 20.

The elementary cross-hatched, four-sided figure in Fig. 20 has sides of length $r\delta\theta$ and δr. Thus, the area of the sector $OACO$ is,

$$A = \sum\sum(r\delta\theta)\delta r = \int_0^{\rho}\int_{\theta_1}^{\theta_2}(rd\theta)dr = \frac{1}{2}\int_{\theta_1}^{\theta_2}\rho^2 d\theta. \quad (29)$$

Here ρ is the distance from the origin 0, i.e. the value of r, to any point P in the curved side of the figure, and the angle θ is in radians.

For a given wedge-shaped figure, the foregoing integration may be performed by any of the practical rules for integration previously described. The quantities ρ^2 and $d\theta$ above are analogous to the quantities y and dx, respectively, in the equation for the area of a typical curvilinear figure in rectangular coordinates; $\overline{OA}$ and $\overline{OC}$ are the end radial distances corresponding to the end ordinates $\overline{OA}$ and $\overline{HG}$ of Fig. 15. The angle AOC is analogous to the length of base $\overline{OH}$ in Fig. 15. The angle AOC is divided by radial lines through O into a suitable number of equal parts. The length of each radial line is squared, and thereafter is treated in the same manner as an ordinate; that is, by applying the proper ordinate multiplier as required by the particular integrating rule that is being used. In polar integration for area, the factor 1/2 that appears before the sign of integration must be used, whereas no such fractional factor exists in front of the integral $\int y dx$ for the area of a figure determined by rectangular coordinates. Also, in polar integration, the common interval is the angular distance in radians between the adjacent radial lines. It is analogous to the linear common interval s of rectangular integration.

In Fig. 20, the centroid of the small elementary triangle POQ is at a distance from O of $(2/3)\rho$. The moment of this elementary triangle about any axis in the plane may be obtained by multiplying its area by the distance of the centroid from that axis. Thus, the distance of the centroid from OY is $(2/3)\ \rho\cos\theta$ and the moment of the elementary triangle POQ about OY is,

$$\frac{1}{2}\rho^2 d\theta \cdot \frac{2}{3}\rho \cos\theta = \frac{1}{3}\rho^3 \cos\theta\, d\theta.$$

The moment M_{oy} of the figure $OACO$ about OY is

$$M_{oy} = \frac{1}{3}\int_{\theta_1}^{\theta_2} \rho^3 \cos\theta\, d\theta. \quad (30)$$

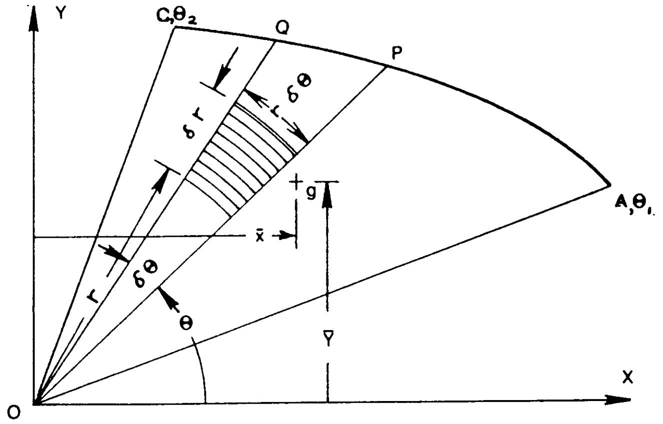


Fig. 20 Curve for integration by polar coordinates

The distance $\bar{x}$ from OY of the centroid, g, of the figure $OACO$ may be obtained by dividing the moment of $OACO$ about OY by the area. Thus distance $\bar{x}$ of g from OY is,

$$\bar{x} = \frac{\frac{1}{3}\int_{\theta_1}^{\theta_2} \rho^3 \cos\theta\, d\theta}{\frac{1}{2}\int_{\theta_1}^{\theta_2} \rho^2\, d\theta}. \quad (31)$$

Table 5—Typical Integrations Using Combined Simpson's First and Second Rules

First Rule as Primary Rule

Sta.	Ordinate	SM	Prod.
0	y_0	1	y_0
1	y_1	4	$4y_1$
2	y_2	2	$2y_2$
3	y_3	4	$4y_3$
4	y_4	17/8	$(17/8)y_4$
5	y_5	27/8	$(27/8)y_5$
6	y_6	27/8	$(27/8)y_6$
7	y_7	9/8	$(9/8)y_7$
			Σ Prod.

Area under curve $= \frac{s}{3} \times \Sigma$ Prod. where s is station spacing.

Second Rule as Primary Rule

Sta.	Ordinate	SM	Prod.
0	y_0	1	y_0
1	y_1	3	$3y_1$
2	y_2	3	$3y_2$
3	y_3	17/9	$(17/9)y_3$
4	y_4	32/9	$(32/9)y_4$
5	y_5	16/9	$(16/9)y_5$
6	y_6	32/9	$(32/9)y_6$
7	y_7	8/9	$(8/9)y_7$
			Σ Prod.

Area under curve $= \frac{3s}{8} \times \Sigma$ Prod., where s is station spacing.

Similarly, the moment of the figure $OACO$ about OX, represented by the symbol M_{ox}, may be written,

$$M_{ox} = \frac{1}{3}\int_{\theta_1}^{\theta_2} \rho^3 \sin\theta \, d\theta. \qquad (32)$$

Also, the distance from OX of the centroid of the entire figure, $\bar{y}$ may be obtained by dividing M_{ox} by the area. Thus distance of g from OX is,

$$\bar{y} = \frac{\frac{1}{3}\int_{\theta_1}^{\theta_2} \rho^3 \sin\theta \, d\theta}{\frac{1}{2}\int_{\theta_1}^{\theta_2} \rho^2 \, d\theta}. \qquad (33)$$

The integration indicated by equations for M_{oy} and M_{ox} may be done in the same way as that previously described for integrating the area equation.

In order to find moment of inertia, about the x-axis for example, it is convenient again to think of the elementary cross hatched portion shown in Fig. 20. Thus, the moment of inertia of the sector $OACO$ about the OX axis is,

$$I_x = \sum\sum (r\delta\theta)\delta r (r\sin\theta)^2$$
$$= \int_O^\rho \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta r^3 dr = \frac{1}{4}\int_{\theta_1}^{\theta_2} \rho^4 \sin^2\theta \, d\theta \qquad (34)$$

In performing a Simpson's Rule integration for moment, or for moment of inertia, based upon polar coordinates, $(1/3)\,\rho^3 \sin\theta$, and $(1/4)\,\rho^4 \sin^2\theta$ respectively would replace the $(1/2)y^2$ and $(1/3)y^3$ terms in similar calculations based upon rectangular coordinates, described in Section 4.2.

4.13 Mechanical Integration. For many years there have been available mechanical instruments allowing the important geometrical properties of any plane curve to be determined without the necessity of reading ordinates and performing a calculation. That is, the final results are obtained directly from dials on the instrument. Unfortunately, the only way to verify the results of such a determination is to repeat the operation. Mechanical integrators are, in effect, a form of analog computer.

The planimeter is used to find the area of any closed curve; the integrator to find the moment of an area about a chosen axis, and sometimes also the moment of inertia. With the integraph the area of any figure may be obtained from its integral curve, which is drawn by the instrument. With a map measurer, the perimeter of any figure, or any part of it may be determined. Of these, the planimeter and the integrator are those most commonly used. Mechanical integration is particularly useful in checking the results obtained by calculations, and also in obtaining quickly, if only approximately, many of the quantities that are needed in the early stages of the design of a vessel.

Today the most common method of calculation is the use of electronic digital computers that employ numerical methods. See section 5.16. Many of the calculations can be performed on a hand-held programmable calculator.

4.14 The Planimeter. The planimeter is an instrument for finding the area of any plane figure. A perspective view of a usual and typical form, known as a polar planimeter[4], is shown in Fig. 21.

The area to be found is bounded by the closed curve $PBEFP$. Any portion of the enclosing line may be straight, curved, or irregular. The planimeter has a tracing point P at one end of a moving bar PA; in operation this tracing point is moved by hand so as to trace entirely around the closed curve. Any point on the curve may be selected from which to start, and the motion is usually in a clockwise direction and continues until the tracing point arrives back at the starting point. The other end of the moving bar PA is jointed at A to a weighted link AO, and this link is free to rotate about the point O. During the operation of the planimeter, the point O, which is located at a needle point on the instrument, is fixed in position on the plane of the table or paper on which the given area is drawn, but O should be outside of the given area.

Attached to the bar PA, and parallel to it, is a shaft on which is mounted the measuring wheel R, which rests on the table or plane of the given figure. The circumferential edge of R is thin so as to insure almost a single-point contact with the horizontal plane. This point of contact, the tracing point P, and the support wheel W constitute the three points of support of the bar BA assembly upon the table. In operation, the

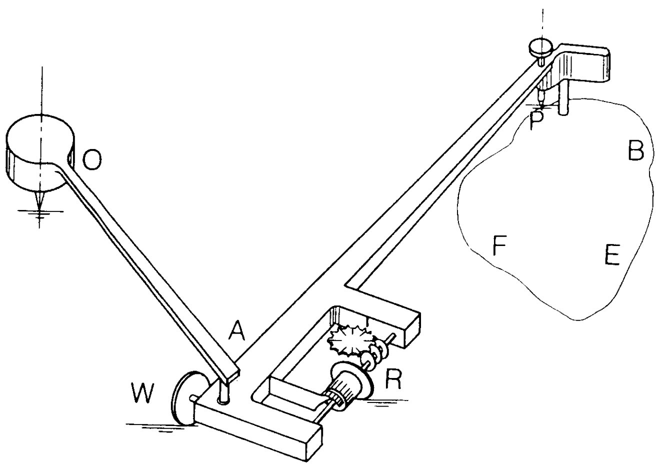


Fig. 21 The planimeter

[4] The planimeter, as well as the integrator (4.15), were invented by Professor Jacob Amsler in Switzerland about 1856.

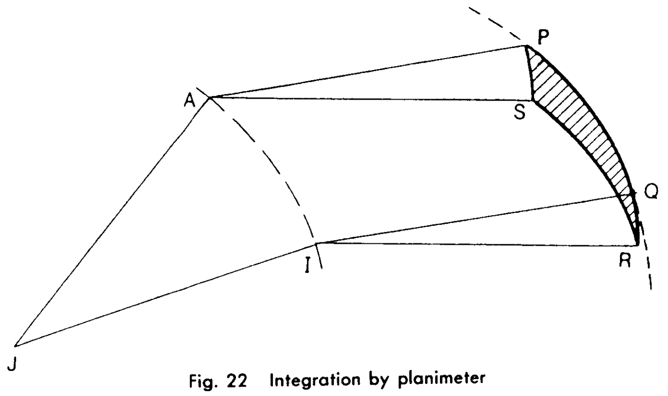


Fig. 22 Integration by planimeter

movement of the assembly can rotate about axis A. The wheel R carries a scale which gives the area reading for the figure $PBEFP$ after it has been traced.

Consider the shaded area $PQRS$ in Fig. 22. Arc PQ is circular, obtained by moving the tracing point at P to Q so that the moving bar IQ remains parallel to AP. Arc QR is circular about I as center and arc SP is circular about A as center. Arc RS is circular, obtained by moving the tracing point from R to S so that the moving bar remains parallel to IR. Since rotation of the wheel results only from motion normal to the moving bar, the increase in revolutions from movement of the tracing point from P to Q is proportional to the area of $APQI$, and the decrease in revolutions in moving from R to S is proportional to the area of $ASRI$. Since sectors APS and IQR are of equal area, tracing arcs QR and SP cancel out, and the cross hatched area $PQRS$ equals the area of $APQI$ minus the area of $ASRI$. Hence, the difference in wheel revolutions before and after tracing right around $PQRS$ is proportional to its area. By approximating any closed curve by an increasingly large number of smaller closed curves generated in the same manner as $PQRS$, we may approach the arbitrary curve as closely as we please, which shows that the area within any closed curve is proportional to the difference in wheel revolutions as a result of tracing right around the curve.

In each actual use it is wise to calibrate the instrument by tracing a rectangle of known area and obtaining a calibration factor.

4.15 The Integrator. The integrator is an instrument for obtaining the area of any plane closed figure and the moment of that area about a chosen axis. In most types of integrators means are also provided for obtaining the moment of inertia of the given area about the same axis. Since it is used primarily for obtaining cross curves of stability, it will be described and discussed in Chapter II.

Section 5

Hydrostatic Curves and Calculations

5.1 Curves of Form. It is customary in the design of a ship to calculate and plot as curves a number of hydrostatic properties of the vessel's form at a series of drafts. Such curves are useful in loading and stability studies during the design phase. Large scale plots of these curves for a newly built ship are then made for the assistance of the vessel's operating personnel. Such curves are known as the vessel's *curves of form*, or synonymously, *hydrostatic curves*. Fig. 23 shows the curves of form for the vessel shown in Fig. 1.

Curves of form are generally drawn on a large sheet of graph paper with all curves plotted against a vertical scale of draft, and with the bottom of the vessel (zero draft) at the foot of the sheet. In order to avoid showing a separate scale for each curve, one horizontal scale of units may be provided, together with separate conversion factors for most of the curves. The practice of providing a horizontal scale of inches, instead of units, often followed in the past, is not recommended in view of the possibility of reproducing the curve sheet at a scale different from that of the original.

Final curves of form as furnished for use by ship's personnel are usually plotted against drafts measured to the bottom of the keel. However, it is not uncommon in the design stage to plot the curves against molded drafts.

The curves of form are customarily calculated with the ship in an even keel condition (no trim). The draft scale is identified as mean draft, and it is assumed that the effect of trim at constant mean draft on most of the plotted quantities is small. This is equivalent to assuming that the vessel is wall-sided—that is, section shapes in way of the prevailing waterline are vertical. The effect of trim is often shown, however, by auxiliary curves.

The range of drafts to which the curves are plotted should extend from below the lightest possible operational draft to the deepest possible draft. The displacement curve should extend down to the origin, in order to provide information for calculating the height of the center of buoyancy, as described in Section 5.10.

5.2 Calculations Required. Calculations of hydrostatic properties of the ship's hull require application of the methods of integration described in Section 4. The calculations take three forms: integrations of

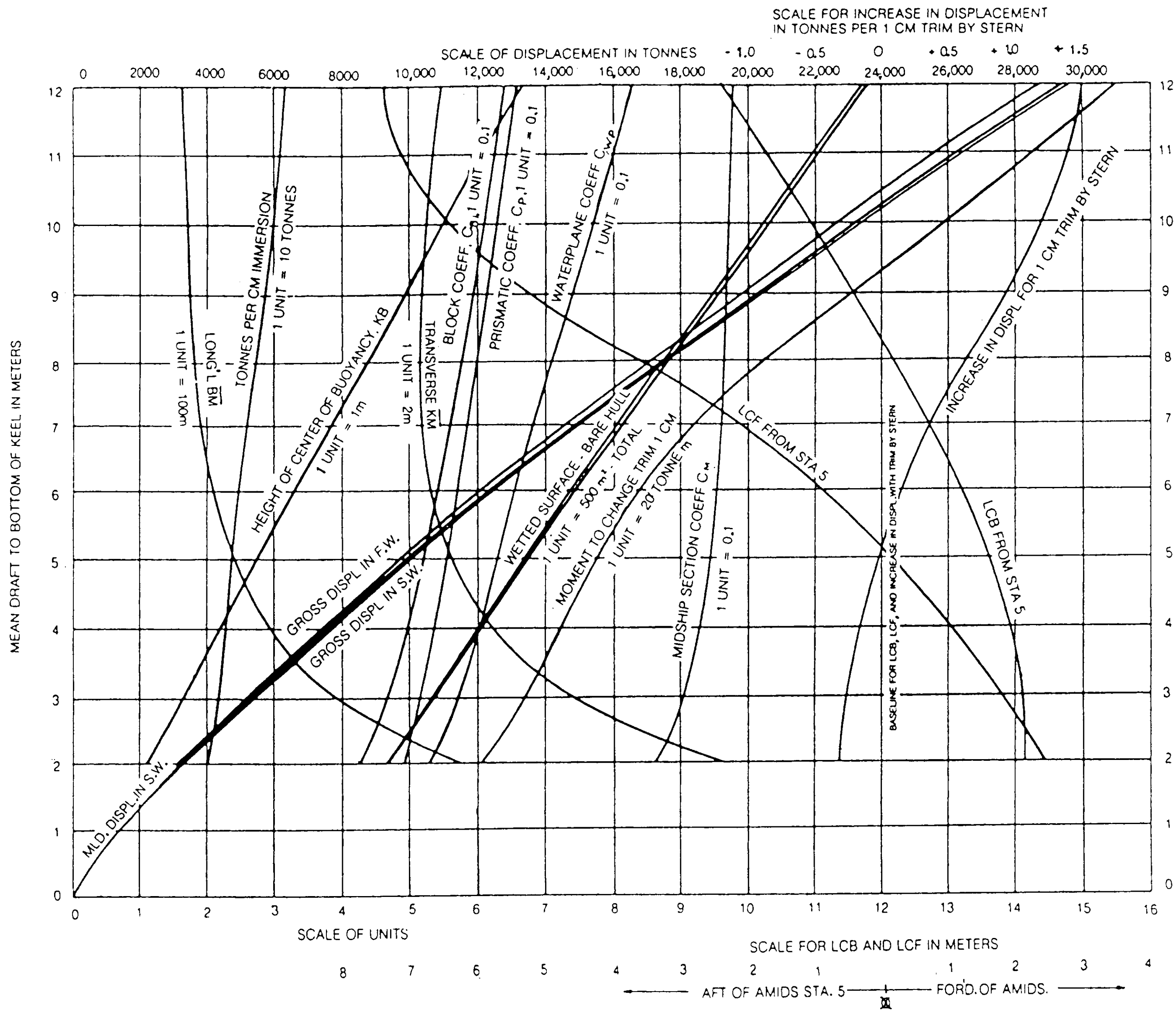


Fig. 23 Curves of form

plane areas to find quantities which are only area related, integrations to find volumes and related quantities, integrations which are accomplished for an area, but for which the answer required also makes use of a volume.

Where calculations for volume of displacement are needed, the integration may be done with either longitudinal distance or vertical distance as the independent variable. Both such integrations are sometimes done for the same volume, to serve as a check.

In all such integrations, the ship's offsets are used. These should always be recorded, and printed out when computer calculations are undertaken.

A formalized tabular method of performing and recording the calculations makes use of a *displacement sheet*, which is a large printed form with spaces for entering all offsets used, multipliers for the rule of integration adopted, and products of these. The displacement sheet forms a permanent record of the calculations, but has the disadvantage of lack of calculational flexibility, and the large sheet may be awkward to file. The majority of such calculations are now accomplished by programmed digital computers. The procedure in this case should include a careful check of the input data, and an assurance that the calculation routines inherent in the computer program are understandable to the user, have adequate precision, and are compatible with the details of the particular form to be integrated.

5.3 Area of Waterplane; Tons per Unit Immersion, Longitudinal Center of Flotation. The ship's waterplane area must be calculated at a sufficiently large number of waterlines to allow a well-defined curve to be drawn over the range of drafts needed. If at any such waterline the draft were to increase by a small amount with no change of trim, the volume of displacement would increase very nearly by an amount equal to the product of waterplane area and the increase in draft, or increase in immersion. The corresponding increase in displacement would be found by multiplying by the density of the water. In the past, an increase in draft of one inch was assumed, giving the *Tons per Inch Immersion* in salt water,

$$TPI = A_{WP}/12 \times 35 = A_{WP}/420 \qquad (35)$$

A preferred quantity in the metric system of measurements is metric *Tons per cm Immersion (TPcm).* Since the density of fresh water is 1 metric ton per m^3, TP cm for a ship in fresh water would be,

$$TP\ cm = A_{WP}/100$$

where A_{WP} is waterplane area in m^2. Assuming the ship in salt water of density, $\rho = 1.025\ t/m^3$,

$$TP\ cm = \rho\ A_{WP}/100 = 1.025\ A_{WP}/100. \qquad (36)$$

With a ship in brackish water, intermediate values of density should be used.

The *Center of Flotation*, which is the point in the waterplane at which a weight added to a vessel would produce parallel sinkage, with no change of trim or heel, is at the centroid of waterplane area. The longitudinal location, LCF, is found by calculating the longitudinal moment of waterplane area in conjunction with the calculation of area. Any axis of reference may be used to find the moment, such as the FP or AP. A midship axis is often preferred, in order to reduce the magnitude of numbers which result. Positive distance is customarily taken forward of amidships; negative

Table 6—Calculation of Waterplane Characteristics at 8.23m (27-ft) Waterline

Station	Half-breadth (m)	½SM	Prod.	Lever	Prod.	$Lever^2$	Prod.	$(Half\text{-}breadth)^3$	Prod.
0	0	0.25	0	5.0	0	25.0	0	0	0
½	1.245	1.0	1.245	4.5	5.603	20.25	25.211	1.93	1.93
1	3.140	0.50	1.570	4.0	6.280	16.0	25.120	30.96	15.48
1½	5.359	1.0	5.359	3.5	18.757	12.25	65.648	153.90	153.90
2	7.597	0.75	5.698	3.0	17.094	9.0	51.282	438.46	328.84
3	10.956	2.0	21.912	2.0	43.824	4.0	87.648	1315.09	2630.18
4	12.007	1.0	12.007	1.0	12.007	1.0	12.007	1731.03	1731.03
5	12.039	2.0	24.078	0	0	0	0	1744.90	3489.80
6	12.039	1.0	12.039	−1.0	−12.039	1.0	12.039	1744.90	1744.90
7	11.899	2.0	23.798	−2.0	−47.596	4.0	95.192	1684.73	3369.46
8	10.271	0.75	7.703	−3.0	−23.109	9.0	69.327	1083.52	812.64
8½	8.417	1.0	8.417	−3.5	−29.460	12.25	103.108	596.31	596.31
9	5.962	0.5	2.981	−4.0	−11.924	16.0	47.696	211.92	105.96
9½	3.057	1.0	3.057	−4.5	−13.756	20.25	61.904	28.57	28.57
10	0	0.25	0	−5.0	0	0	0	0	0
			$\Sigma_1 = 129.864$		$\Sigma_2 = -34.319$		$\Sigma_3 = 656.182$		$\Sigma_4 = 15009.00$

Station sp., $s = \frac{L}{10} = \frac{154.99}{10} = 15.499$ m

Waterplane area, $A_{WP} = \Sigma_1 \times \frac{4}{3} \times s = (129.864 \times 20.666) = 2{,}683.77\ m^2$

Waterplane coeff., $C_{WP} = A_{WP}/(L \times B) = 2683.77/(154.99 \times 24.078) = 0.719$

Tonnes per cm immersion = 2,683.77 × 1.025/100 = 27.51 t (S.W.)

Long'l Center of Flotation $LCF = (\Sigma_2/\Sigma_1) \times s = (-34.319/129.864) \times 15.499 = 4.10$ m abaft Sta. 5

Long'l moment of inertia about Sta. 5 $= \Sigma_3 \times \frac{4}{3} \times s^3 = 656.182 \times \frac{4}{3} \times (15.499)^3 = 3{,}257{,}400\ m^4$

Long'l moment of inertia about LCF, $I_L = 3{,}257{,}400 - 2{,}683.77 \times (4.10)^2 = 3{,}212{,}300\ m^4$

Trans. moment of inertia, $I_T = \Sigma_4 \times \frac{4}{9} s = 15{,}009 \times 6.8884 = 103{,}390\ m^4$

Vol. of displacement, ∇ (from displacement curve) = 17,845 m^3

Long'l $BM = I_L/\nabla = 3{,}212{,}300/17{,}845 = 180.0$ m

Transverse $BM = I_T/\nabla = 103{,}390/17{,}845 = 5.79$ m.

is abaft amidships. LCF is then moment divided by area.

Table 6 shows the tabular calculation for these quantities. The rule of integration used is Simpson's First Rule with half multipliers; the integrating factor for area, in accordance with section 4.2 (b) is $2 \cdot 2 \cdot \frac{s}{3}$. The results apply to the full waterplane (both sides of ship). The integrating factor for longitudinal moment is $2 \cdot 2 \cdot \frac{s^2}{3}$, since each ordinate is weighted by its dimensionless distance from amidships, in units of station spacing, s. Also included with Table 6 is the calculation for waterplane coefficient C_{WP}, discussed in Section 3.

$$C_{WP} = \frac{\textit{Waterplane area}}{L \cdot B}.$$

Here L is the length between perpendiculars, although the actual length of the example waterplane exceeds LBP. Both the TPcm and LCF curves are useful for checking the correctness of input data, in that errors in offsets used in calculating these two curves are usually detectable in the uncharacteristic appearance of the curves.

5.4 Transverse Metacentric Radius; Height of Transverse Metacenter. The terms *transverse metacenter* and *transverse metacentric height* are defined and discussed in Chapter II. There it is shown that the vertical distance from the center of buoyancy to the transverse metacenter is called transverse metacentric radius $\overline{BM}$, where

$$\overline{BM} = I_T / \nabla, \qquad (37)$$

and I_T is transverse moment of inertia of entire waterplane area about the longitudinal centerline and ∇ is volume of displacement. Molded dimensions and volume are customarily used in this calculation (Section 5.7).

Table 6 includes the calculation for I_T, inasmuch as the waterplane halfbreadths needed are available from the waterplane area calculation. The integrating factor is the same as for waterplane area, but multiplied by 1/3, and halfbreadths must be cubed in accordance with Section 4.2.

Also shown with Table 6 is the calculation of $\overline{BM}$. Here the volume of displacement ∇ may be obtained by displacement calculations up to the same waterline, or may be read from the molded displacement curve.

The height of the transverse metacenter above the molded baseline is called $\overline{KM}_T$, or simply $\overline{KM}$. This is found by adding the height of the center of buoyancy $\overline{KB}$ to the metacentric radius $\overline{BM}$. That is,

$$\overline{KM} = \overline{KB} + \overline{BM}. \qquad (38)$$

The curves of form include transverse $\overline{KM}$, which is an important quantity for a ship from considerations of stability. It is important to distinguish between the height of metacenter $\overline{KM}$, which is a purely geometrical quantity, and the metacentric height, $\overline{GM}$, which involves the location of the ship's center of gravity, as discussed in Chapter II.

5.5 Longitudinal Metacentric Radius; Height of Longitudinal Metacenter. The terms *longitudinal metacenter* and *longitudinal metacentric height* are also discussed and defined in Chapter II. Longitudinal metacentric radius is there defined as $\overline{BM}_L$, where

$$\overline{BM}_L = I_L / \nabla, \qquad (39)$$

and I_L is longitudinal moment of inertia of entire waterplane area about a transverse axis through the longitudinal center of flotation LCF.

Table 6 also includes the calculation for longitudinal moment of inertia of the waterplane about amidships, which requires the waterplane halfbreadths. In performing the calculation, each ordinate must be weighted by the square of its distance from the reference axis. This is done nondimensionally in units of the common interval, or station spacing s. The integrating factor is,

$$2 \cdot 2 \cdot s^3/3.$$

In order to correct the longitudinal moment of inertia to a transverse axis through the LCF, the product of $A_{WP} \cdot (\text{LCF})^2$ is deducted from the longitudinal moment of inertia about amidships, in accordance with section 4.2. This calculation is shown at the foot of Table 6.

The longitudinal metacentric radius $\overline{BM}_L$ is then calculated and the height of the longitudinal metacenter above the baseline $\overline{KM}_L$ is found, where

$$\overline{KM}_L = \overline{KB} + \overline{BM}_L. \qquad (40)$$

As in the case of the transverse metacenter, the height of the longitudinal metacenter $\overline{KM}_L$ should not be confused with *longitudinal metacentric height*, which is discussed in Chapter II.

5.6 Molded Displacement and Total Displacement. The displacement of a vessel is the product of underwater volume—or volume of displacement—and the density of the medium in which the vessel floats. Curves of form for an oceangoing vessel usually include three displacement curves; molded displacement in salt water, total or gross displacement in salt water, and total displacement in fresh water, Fig. 23. Of these, total displacement in salt water is probably the most useful to operating personnel of oceangoing ships. A scale of displacement in metric tons is usually provided at the top of the curve sheet.

The volume of the underwater portion of a steel

Table 7—Calculation of Displacement and Longitudinal Center of Buoyancy at 5m (16.41-ft) Waterline

Station	Area $(m)^2$	½ SM	Prod	Lever (nondimens.)	Prod	
−0.07	0	0.0175	0	5.07	0	First Rule (−0.07 to 0)
−0.035	3.0	0.07	0.21	5.035	1.1	
0	4.2	0.2675	1.12	5.0	5.6	
½	12.7	1.00	12.70	4.5	57.2	
1	22.6	0.50	11.30	4.0	45.2	
1½	35.1	1.00	35.10	3.5	122.9	
2	50.6	0.75	37.95	3.0	113.9	
3	83.3	2.0	166.60	2.0	333.2	First Rule (0 to 8)
4	106.1	1.0	106.10	1.0	106.1	
5	113.7	2.0	227.40	0	0	
6	107.6	1.0	107.60	−1.0	−107.6	
7	81.4	2.0	162.80	−2.0	−325.6	
8	44.0	0.7813	34.38	−3.0	−103.1	
8½	29.1	0.8438	24.30	−3.5	−85.1	Second Rule (8 to 9)
9	17.4	0.8438	14.68	−4.0	−58.7	
9½	5.3	0.3138	1.66	−4.5	−7.5	First Rule (9 to 9.63)
9.565	3.3	0.13	0.43	−4.57	−2.0	
9.63	0	0.0325	0	−4.63	0	
			Σ_1=944.33		Σ_2=95.6	

Sectional area curve extended beyond Stations 0 and 9½ to extremities, as shown by Fig. 24 and read at midpoint between last station and extremity. Simpson's Multipliers proportioned accordingly. Thus, at Station −0.035, ½ SM = ½ × 4 × 0.035 = 0.07; at Station 0, ½ SM = 0.25 + 0.0175 = 0.2675; at Station 8, ½ SM = $\frac{1}{2}\left(1.0 + \frac{9}{8} \times \frac{1}{2}\right)$ = 0.7813 (First and Second Rules); at Station 8½ and 9, ½ SM = $\frac{1}{2} \times \frac{9}{8} \times \frac{3}{2}$ = 0.8438; at Station 9½, ½ SM = $\frac{1}{2}\left(\frac{9}{8} \times \frac{1}{2} + \frac{0.065}{1.0}\right)$ = 0.3138; at Station 9.565, ½ SM = ½ × 4 × $\frac{0.065}{1.0}$ = 0.13.

Then ∇, volume of displacement = $\Sigma_1 \times \frac{2}{3} \times s = 944.33 \times \frac{2}{3} \times 15.499 = 9{,}757\ m^3$.

Displacement, $\Delta = 1.025 \times 9757 = 10{,}000$ t (SW.)

$$LCB = \frac{\Sigma_2}{\Sigma_1} \times s = \frac{95.6}{944.33} \times 15.499 = 1.57 \text{ m for'd of Station 5.}$$

vessel is made up of the volume of the molded form (Section 5.7) plus the volume of the steel shell plating and other appendages, such as rudder, propeller, shaft bossings, sonar domes, bilge keels, etc. (Section 5.13). In a wooden vessel it is the volume to the outside of planking plus the volume of other appendages.

In a steel single-screw cargo vessel the volume of all appendages is usually slightly less than 1 percent of the molded volume, and the shell plating is by far the largest contributor, perhaps 0.75 percent. For very large ships, this percent tends to be lower—less than 0.5 percent for a typical large tanker. In multiple-screw vessels, the appendages constitute a greater percentage of the molded volume than in the case of single-screw vessels.

Of the various curves of form, the displacement curves are of particular importance. They are expected to be accurate, and are frequently utilized for the precise determination of displacement, as for example at the inclining experiment and deadweight check before delivery of a vessel.

Having calculated the displacement at different drafts, the coefficients of form discussed in Section 3 can be readily calculated. Curves of c_B, c_P, c_M, and c_{WP} are usually included among the curves of form.

5.7 Displacement and LCB. The calculation of a molded displacement curve requires that all portions of the vessel below the waterline of interest be included. This requires integration of volumes upward from the baseline. Should the vessel extend below the baseline, as from drag to the keel, a finite volume of displacement would exist at zero mean molded draft. The usual method is to calculate sectional areas directly and then to integrate them longitudinally. The longitudinal center of buoyancy for the waterline of interest is also conveniently found in this calculation.

Table 7 shows such a calculation up to one waterline for the example ship of Fig. 1. Simpson's First Rule

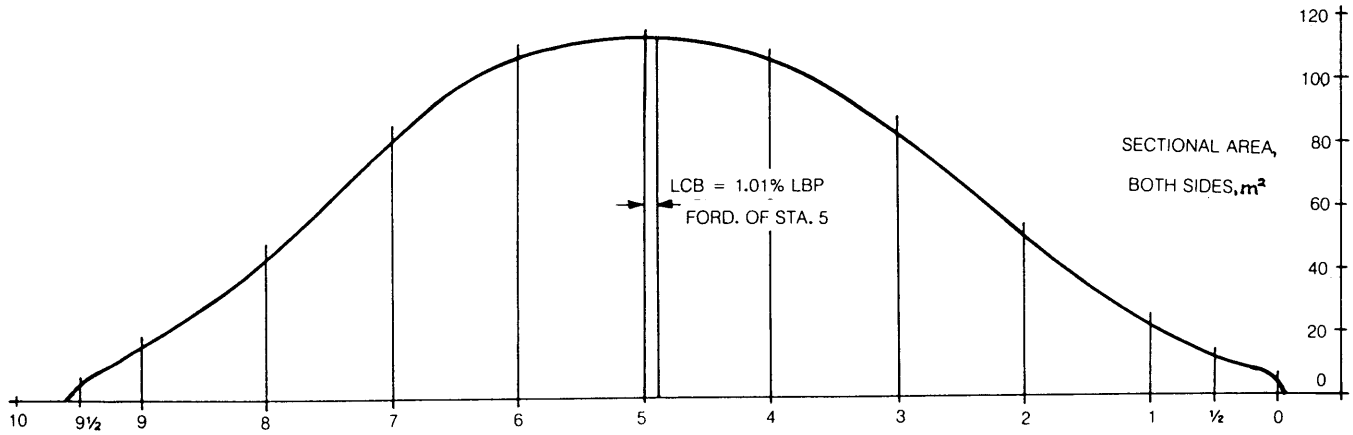


Fig. 24 Sectional area curve for 5 m waterline

is used as the primary rule, with half multipliers. In order to find the longitudinal moment of volume, the sectional areas are multiplied by their non-dimensional distances from amidships. The longitudinal center of buoyancy is then found as the quotient of moment divided by volume. In some cases the inclusion of appendages may have a significant effect.

The integrating factor for volume of displacement is simply $2 \cdot s/3$, inasmuch as the sectional areas used represent the total sectional area for both sides of the ship, as plotted for *Bonjean Curves*. In the case of longitudinal moment, the integrating factor is $2 \cdot s^2/3$. Here s is station spacing.

A substantial number of points are desired for the displacement curve. Most vessels change rapidly with draft at the lower waterlines. For example, a ship with small deadrise and no drag would have a large change in TP cm between zero mean draft and say the 0.5 m waterline.

In order to find the area to the lowest waterline one may use a planimeter, or else a number of closely-spaced waterlines together with numerical integration. The latter method is often used with digital computer calculations.

Calculations of volume and moment of volume for upper waterlines may be simplified somewhat if they are confined to successive "layers" of the underwater body above the uppermost waterline for which volume and moment calculations have already been completed. Thus, to calculate values up to the 5 m waterline, the "ordinate" to the curve to be integrated may be taken as the difference between areas for 5 m and 4 m waterlines. The volume of displacement increase so found would be added to that for the 4 m waterline.

As shown in Section 2.2, displacement is obtained from displacement volume by multiplying by the mass density of the liquid in which the ship is assumed to float. In SI units,

$$\Delta = \rho \nabla. \quad (2)$$

In English units,

$$W = \rho g \nabla \text{ tons.} \quad (1)$$

Correspondingly, to find LCB for the 5-m waterline, the longitudinal moment of volume between the 5 m and 4 m waterlines may be added to that for the 4 m waterline. LCB for the 5 m waterline is obtained by dividing total moment by total volume.

Fig. 24 shows the sectional area curve for the 5 m waterline, as well as its longitudinal centroid, which represents LCB at a molded draft of 5 m without trim.

In principle, waterplane areas may be found at several closely spaced, but low waterlines, and these areas integrated vertically to find displacement. However, greater accuracy is generally attainable by the longitudinal integration method.

Having calculated the displacement at different drafts, the coefficients of form discussed in Section 3 can be readily calculated.

5.8 Vertical Center of Buoyancy by Vertical Integration of Waterplanes. A basic feature of any vessel from the point of view of stability is the height of the center of buoyancy above the baseline, called $\overline{KB}$. It may be calculated by first finding the vertical moment of the volume of displacement above the baseline at any waterline.

In integral form, the moment up to draft T_P is,

$$\int_o^{T_P} T A_{WP} dT.$$

$$\text{Then } \overline{KB} = \frac{\int_o^{T_P} T A_{WP} dT}{\int_o^{T_P} A_{WP} dT} = \frac{\int_o^{T_P} T A_{WP} dT}{\nabla}. \quad (41)$$

The calculation requires a curve of waterplane areas vs. draft, as shown in Fig. 25. The vertical location of the centroid of the area above this curve and below the waterline is identical with $\overline{KB}$ for any given draft.

For lower waterlines, a combination of the 5,8,-1 rule, giving volume of displacement, and the 3,10,-1 rule, giving moment of volume, may be utilized for integration, on the assumption that the plot of waterplane area against draft resembles a parabolic curve of the second order. Table 8 shows such a calculation for that portion of the vessel in Fig. 1 below the 1 m waterline. The integrating factor for volume of displacement is $s/12$ where s is waterline spacing. The integrating factor for moment of volume is $s^2/24$.

The same general procedure may be used for upper waterlines, but with the calculated volume and moment of volume, for the appropriate pairs of waterlines, added to corresponding values for the waterline below. Table 8 includes the calculations of volume of displacement and $\overline{KB}$ up to the 2 m waterline.

5.9 Vertical Center of Buoyancy by Integration of Displacement Curve. Another method of obtaining $\overline{KB}$ depends upon the fact that the curve of volume of displacement vs. draft is the integral of the curve of waterplane areas. Fig. 26 shows a curve of volume of displacement ∇ against draft T. The area A of the cross hatched section between the curve and the horizontal line at the prevailing draft T_p may be expressed as the sum of many small rectangles of area, $\delta\nabla(T_p - T)$. Thus,

Table 8—Calculation of Volume of Displacement and Height of Center of Buoyancy by Vertical Integration of Waterplane Areas

Height above baseline, m	Waterplane area, m²	Multiplier for volume	Product	Multiplier for moment	Product
0	194	5	970	3	582
1	1714	8	13712	10	17140
2	1976	−1	−1976	−1	−1976
			Σ_1=12706		Σ_2=15746
1	1714	5	8570	3	5142
2	1976	8	15808	10	19760
3	2137	−1	−2137	−1	−2137
			Σ_3=22241		Σ_4=22765

Values for 1 m draft

$$\text{Volume of displ., } \nabla = \frac{S}{12} \times \Sigma_1 = \frac{1}{12} \times 12{,}706 = 1059 \text{ m}^3$$

$$\text{Moment of volume about baseline, } M_\nabla = \frac{S^2}{24} \times \Sigma_2$$

$$= \frac{1^2}{24} \times \Sigma_2 = \frac{1^2}{24} \times 15{,}746 = 656.1 \text{ m}^4$$

$$\text{Height of center of buoyancy, } \overline{KB} = \frac{M_\nabla}{\nabla} = \frac{656.1}{1059} = 0.62 \text{ m.}$$

Values for 2 m draft

$$\text{Added volume of displ., } \delta\nabla \text{ (1 m to 2 m)} = \frac{s}{12} \times \Sigma_3 = \frac{1}{12} \times 22{,}241 = 1853 \text{ m}^3$$

$$\text{Total volume, } \Sigma\nabla = \nabla + \delta\nabla = 1059 + 1853 = 2912 \text{ m}^3$$

Moment of added volume, δM_∇ (1 to 2 m) about 1 m waterline

$$= \frac{s^2}{24} \times \Sigma_4 = \frac{1^2}{24} \times 22{,}765 = 948.5 \text{ m}^4$$

$$\text{Moment of added volume about baseline} = 948.5 + 1853\,(1 - 0) = 2801.5 \text{ m}^4$$

$$\text{Moment of total volume about baseline, } \Sigma M_\nabla = 656.1 + 2801.5 = 3457.6 \text{ m}^4$$

$$\text{Height of center of buoyancy, } \overline{KB} = \frac{\Sigma M\nabla}{\Sigma\nabla} = \frac{3457.6}{2912} = 1.19 \text{ m.}$$

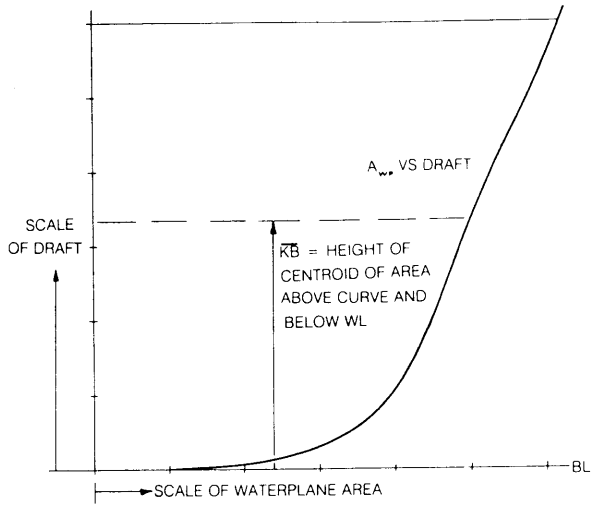


Fig. 25 Waterplane area vs. draft

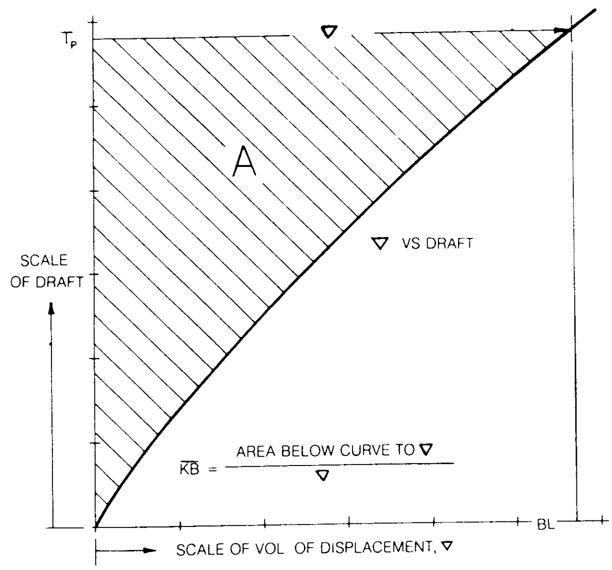


Fig. 26 Volume of displacement vs. draft

$$A = \sum(T_P - T)\delta\nabla = \int_o^{Tp} (T_P - T)d\nabla \text{ in the limit.}$$

But the volume of displacement $\nabla = \int_o^T A_{WP}dT$ and $d\nabla = A_{WP}dT$, where A_{WP} is waterplane area.

Hence, separating the integral into two parts and substituting for $d\nabla$,

$$A = \int_o^{Tp} (T_p - T)d\nabla = T_p\int_o^{Tp} d\nabla - \int_o^{Tp} Td\nabla$$
$$= T_p\int_o^{Tp} A_{WP}dT - \int_o^{Tp} A_{WP}TdT.$$

The first of these integrals is volume of displacement up to T_p. The second integral represents the moment of the volume of displacement about the baseline $M_{\nabla,o}$ or,

$$A = T_p\nabla - M_{\nabla,o}, \text{ and } M_{\nabla,o} = T_p\nabla - A.$$

Inasmuch as the area of the rectangle formed by T_p and ∇ at T_p is simply $T_p \cdot \nabla$, we see that to find the moment of the volume of displacement about the baseline, it is merely necessary to find the cross hatched area A and deduct it from the product of $T_p \cdot \nabla$. Alternatively, one can integrate the un-crosshatched area *under* the curve directly. The vertical center of buoyancy $\overline{KB} = \frac{M_{\nabla,o}}{\nabla}$ by definition, and therefore,

$$\overline{KB} = \frac{T_p\nabla - A}{\nabla} \quad (42)$$

This provides a simple way of finding the vertical height of the center of buoyancy at any draft. In practice, a displacement curve must be available extending right to the baseline. The procedure to follow is to draw a vertical line at the desired displacement and carefully measure the area between the base line and the curve by a rule of integration or by planimeter. Then dividing the measured area by the displacement gives $\overline{KB}$.

It may be noted that the ton scale to which the displacement curve is usually plotted presents no problem here, inasmuch as the scale factor relating Δ to ∇ cancels when taking the quotient.

This method is considered the most accurate for finding $\overline{KB}$ at low drafts, but requires that the displacement curve at its lower end be carefully defined. Using longitudinal integration as described in Sec. 5.7, finding precise values of displacement at low waterlines should not pose a problem.

5.10 Approximate Formulas for Vertical Center of Buoyancy. In the initial stages of design, the height of the center of buoyancy may be required, yet a displacement curve is not available, precluding a calculation of $\overline{KB}$ by the method of Section 5.9. To this end, approximate formulas may be used as given below.

The Morrish formula (Morrish, 1892), also known as Normand's formula, gives the distance below the DWL of the center of buoyancy as,

$$\frac{1}{3}\left(\frac{T}{2} + \frac{\nabla}{A_{WP}}\right),$$

where T is molded mean draft, ∇ is corresponding volume of displacement and A_{WP} is corresponding waterplane area, all in a consistent system of units. The expression may be written more directly as,

$$\overline{KB} = \frac{1}{3}\left(\frac{5T}{2} - \frac{\nabla}{A_{WP}}\right). \quad (43)$$

Experience has shown that for vessels of ordinary form the formula gives close approximation to the height of the center of buoyancy, not only for load draft but also for lighter drafts.

The formula may be derived from a diagram such as Fig. 27 in which *KUP* is a curve of waterplane areas plotted against molded drafts. *LP* equals A_{WP}, waterplane area at mean molded draft *T*, which equals *KL*. The area of the figure *LKUPL* is equal to ∇, volume of displacement, and the centroid of this figure is at the same height above the baseline as the center of buoyancy. It is assumed that the centroid of the polygon *LKBPL* is at the same height as the centroid of *LKUPL*.

Another approximation which gives values quite close to those of Fig. 23 is Posdunine's formula (Posdunine, 1925),

$$\overline{KB} = T\left(\frac{A_{WP}}{A_{WP} + \frac{\nabla}{T}}\right). \quad (44)$$

The symbols have the same meaning as for the previous expression.

Experience with this formula indicates particularly good agreement with ships of high midship coefficient, C_M.

5.11 Change of Displacement with Trim. Curves of form are customarily calculated for an even keel (no trim) condition. It is shown in Chap II that when a vessel changes trim by a moderate amount because of the movement of a weight forward or aft, the draft remains constant at the center of flotation. Thus, if the center of flotation is abaft amidships, a weight shift causing trim by the stern will result in a reduction in mean draft. Therefore, if it be stipulated that mean draft remain constant, an increase in displacement results from trim by the stern in this case. The approximate increase in displacement in t per cm, (or tons per in.), of trim by the stern is (see Chapter II, 8.6),

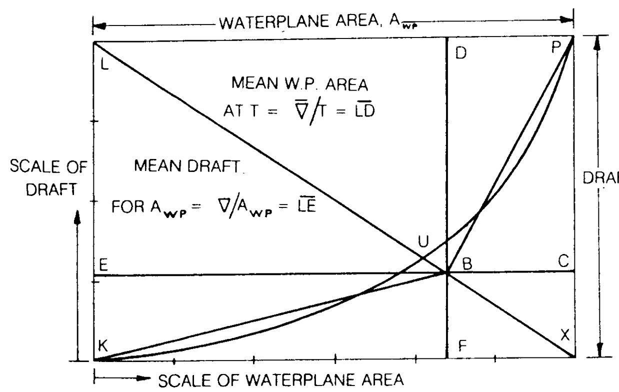


Fig. 27 Morrish Formula diagram

$$TPcm\ d/L \quad (45)$$

or

$$TPI\ d/L \quad (46)$$

where

d = distance LCF is abaft amidships,

L = length of ship between draft marks.

It is important that the direction of trim, by bow or stern, and the sign of the change in displacement, be clearly labeled on the curve.

As one of the curves of form, *change of displacement with trim* usually shares with longitudinal center of buoyancy, and longitudinal center of flotation, a separate reference axis from the displacement curve. If the *LCF* curve crosses amidships, the value of the change of displacement curve will be zero at the draft at which it crosses.

5.12 Moment to Change Trim. The moment necessary to change trim by a fixed quantity is an important characteristic of a vessel and one frequently used for loading studies. The *Moment to Change Trim 1 cm* (*MTcm*) may be found using principles outlined in Chap II. The expression is,

$$MTcm = \frac{\Delta \cdot \overline{GM}_L}{100\ \mathrm{L}}, \quad (47)$$

where Δ is ship displacement in metric tons, $\overline{GM}_L$ is longitudinal metacentric height = $\overline{KM}_L - \overline{KG}$ in m, *L* is length of ship between draft marks in m.

In English units, long tons and ft,

$$MT1'' = \frac{W\overline{GM}_L}{12L}. \quad (48)$$

The value of $\overline{KM}_L$ is found as noted in Sect. 5.5 for any draft. The height of center of gravity $\overline{KG}$ will depend upon the loading condition of the ship. However, for most ships, the range of values of $\overline{KG}$ to be expected in service is a relatively small percentage of $\overline{GM}_L$ and it is sufficiently accurate to assume a standard and reasonable location. For many ships the height of the center of gravity is not far from the prevailing draft, and that location is sometimes chosen for the curves of form. However, for the example ship, the height of the center of gravity is assumed to be the same as the height of the center of buoyancy, so that $\overline{GM}_L = \overline{BM}_L$.

Inasmuch as it is directly proportional to displacement, it is evident that *MTcm* will vary directly with water density. Thus, at any draft *MTcm* should increase when in salt water, compared with its value in fresh water.

5.13 Displacement of Appendages. In order to find the total displacement of a ship, the displacement of the appendages—shell plating, rudder, propellers, bilge keels, bossings, etc.—up to any given waterline must be calculated and added to the displacement of

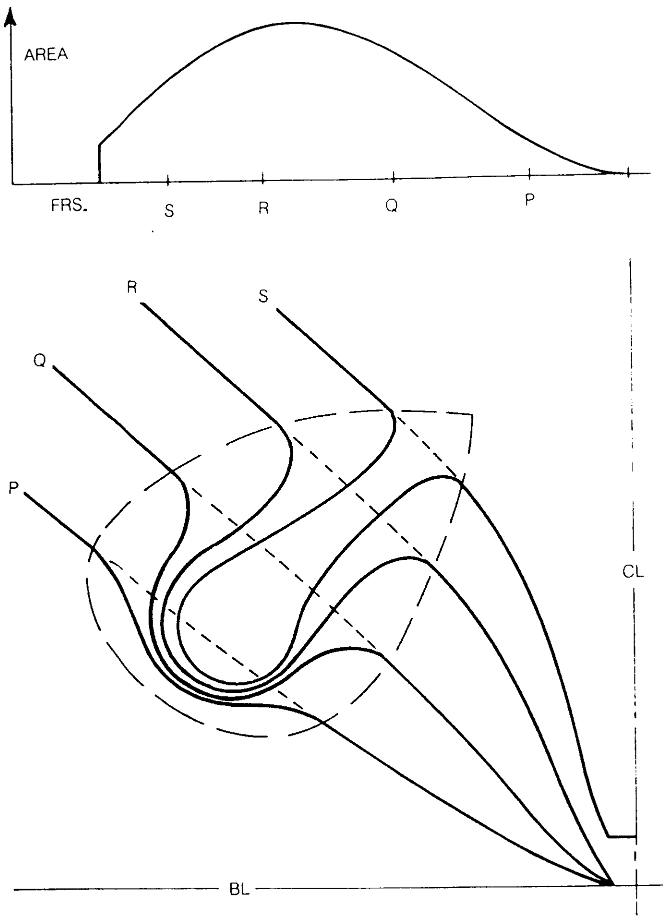


Fig. 28 Sections and sectional area curve of bossing

the molded form. The displacement of these appendages may be calculated from the shell expansion, midship section, and detail drawings of the various parts.

A typical example of such an appendage is the bossing on a twin screw ship. Fig. 28 shows transverse sections through such a bossing and the longitudinal distribution of areas associated with it, or sectional area curve. As in the case of the ship's molded form, the area of the sectional area curve represents the volume of displacement of the bossing.

Many ships built in recent years are fitted with transverse thrusters, which take the form of a cylindrical transverse free-flooding tunnel through the ship at a low waterline. These represent departures from the molded form of the ship, and may be accounted for as a "negative" appendage—that is, the net free-flooding volume of the tunnel should be deducted from the volume of other appendages. In case of naval ships with large sonar domes, the dome may be treated as part of the molded form, or alternatively as an appendage. The convention adopted should be clearly stated on the curve sheet.

In calculating the displacement of the shell plating, any strakes of "out" plating must be properly accounted for. For example, if the flat plate keel is an "out" strake, the volume between its inner surface and the molded form is also treated as appendage volume.

5.14 Curves of Form for Particular Types of Vessels. Certain specialized vessels may call for additional curves of form beyond those discussed in the foregoing. For example, bulk carrying vessels such as tankers and ore carriers can experience more than insignificant degrees of hull girder bending in still water as a result of concentrated weights. If the deflection of the hull girder can be predicted, this can be accounted for in hydrostatic characteristics, such as displacement for drafts read at bow and stern.

A simple but not unrealistic hull girder deflection curve is a second order parabola $y = ax^2$ where y is the deflection from the zero bending moment case, x is longitudinal distance from amidships, non-dimensionalized by the length of the ship, and a is vertical deflection at the ends of the ship, resulting in hog or sag, compared with amidships.

To find the increase in displacement with sag, waterplane halfbreadths must be weighted, first by the deflection, taken as $(a - y)$, and then by the rule of integration multipliers (Simpson's multipliers). Table 9 shows diagrammatically how the calculation may be performed, the increase in displacement with sag being that to be added to the displacement from drafts read at the ends of the vessel, assuming a straight keel.

In case the effect of trim on displacement is considered to be of unusual importance, trim correction curves may be calculated by assuming a series of trimmed waterlines, say for 1 m trim, 2 m trim, etc. by both bow and stern, and at a series of drafts. Such calculations are facilitated by the use of Bonjean curves, as discussed in Section 6. Resulting trim corrections in displacement are more accurate than can be obtained by the change in displacement with trim curve described in Section 5.11, inasmuch as they do not assume the vessel is wall-sided.

Unusual hydrostatic properties may be found for particular types of vessels, including floating drydocks, offshore mobile platforms, integrated tug-barges, and ships with large compartments which are occasionally free flooded, such as the dock area on float on-float off barge carriers. The curves of form in these cases may be characterized by knuckles at the draft at which large elements of buoyancy are immersed, or by several curves of the same quantity, depending upon the ship's condition. Careful thought is needed in analyzing such vessels. However, the calculation of displacement, and displacement related curves, may be more directly done when the underwater body is largely composed of simple geometrical bodies—cylinders, cones, prisms—rather than surfaces of compound curvature.

Table 9—Calculation for Finding Increased Displacement per Meter Sag

Sta.	Dimensionless distance from amidships (x/L)	Deflection, y m	($y_{max} - y$) m	HB m	SM	Product
0	0.5	1.00	0	y_0	½	0
½	0.45	0.81	0.19	$y_{1/2}$	2	0.38 $y_{1/2}$
1	0.4	0.64	0.36	y_1	1	0.36 y_1
1½	0.35	0.49	0.51	$y_{1\frac{1}{2}}$	2	1.02 $y_{1\frac{1}{2}}$
2	0.3	0.36	0.64	y_2	3/4	0.48 y_2
3	0.2	0.16	0.84	y_3	4	3.36 y_3
4	0.1	0.04	0.96	y_4	2	1.92 y_4
5	0	0	1.00	y_5	4	4.00 y_5
6	−0.1	0.04	0.96	y_6	2	1.92 y_6
7	−0.2	0.16	0.84	y_7	4	3.36 y_7
8	−0.3	0.36	0.64	y_8	3/4	0.48 y_8
8½	−0.35	0.49	0.51	$y_{8\frac{1}{2}}$	2	1.02 $y_{8\frac{1}{2}}$
9	−0.4	0.64	0.36	y_9	1	0.36 y_9
9½	−0.45	0.81	0.19	$y_{9\frac{1}{2}}$	2	0.38 $y_{9\frac{1}{2}}$
10	−0.50	1.00	0	y_{10}	½	0
						Σ

Assumed deflection, $y = 4 \times \left(\frac{x}{L}\right)^2$

Increased volume of displacement per meter sag $= \frac{2}{3} \times s \times \Sigma$ in m³ (both sides of ship), where s is station spacing in m.

Table 10—Condensed Summary of Curves of Form Values

Mean draft to bottom of keel, meters	Displacement in metric tons: Molded in salt water	Gross in salt water	Gross in fresh water	LCB, long'l. center of buoyancy from ⊗, m	LCF, Long'l. center of flotation from ⊗, m	$\overline{KB}$, Center of buoyancy above baseline, m	Tons per cm immersion	Moment to change trim 1 cm, t-m
2	3243	3273	3151	2.15 F	2.41 F	1.11	20.3	121
3	5347	5418	5245	2.11 F	1.83 F	1.65	21.9	134
5	9941	10033	9748	1.59 F	0.08 F	2.74	24.0	156
7	14932	15065	14658	0.73 F	2.11 A	3.82	26.0	184
9	20350	20462	19903	0.42 A	5.20 A	4.93	28.9	232
11	26317	26449	25707	1.73 A	7.08 A	6.08	30.8	286
12	29448	29611	28757	2.32 A	7.38 A	6.64	31.8	311

Mean draft to bottom of keel, meters	Increase in displacement for 1 cm trim by stern, t	$\overline{KM}$, Transv. metacenter above baseline m	$\overline{BM}_L$, Long'l. metacentric radius m	C_B Block coeff.	C_M Midship coeff.	C_P Prismatic coeff.	C_W Waterplane coeff.	Wetted surface in square meters: Bare Hull	Total
2	−0.306	19.4	579	0.427	0.864	0.494	0.529	2332	2339
3	−0.260	14.8	388	0.470	0.908	0.518	0.573	2669	2685
5	−0.023	11.3	243	0.522	0.944	0.553	0.629	3344	3377
7	+0.367	10.4	190	0.562	0.961	0.584	0.680	4013	4066
9	+0.951	10.4	177	0.593	0.969	0.612	0.743	4733	4794
11	+1.407	10.7	169	0.626	0.976	0.642	0.803	5478	5540
12	+1.507	11.0	164	0.641	0.978	0.658	0.830	5852	5920

Principal Ship Dimensions

Length overall, m	166.60	Design draft, molded, m	8.23
Length between perpendiculars, m	154.99	Displacement, molded, at design draft, s.w., tons	18,250
Length for coefficients, m	154.99	Bottom of keel below baseline, m	0.0254
Breadth, molded, m	24.08	Half siding at baseline, m	0.660
Depth, molded, to main deck amidships, m	14.66	Deadrise, m	0.305

1. Calculate, plot Bonjean curves (see Section 6).
2. Read Bonjean curves at desired drafts; integrate longitudinally to calculate, then plot at each draft: displacement, *LCB;* extend displacement curve to zero draft.
3. Read waterline half-breadths at each draft desired, calculate waterplane area, calculate, plot *LCF,* TPcm, C_{WP}; calculate I_T about ships centerline, calculate I_L about transverse axis through *LCF* at each draft.
4. Using displacement from (2) and I_T and I_L from (3), calculate, plot BM_L; calculate $\overline{BM}$, all at each draft.
5. Integrate displacement curve vertically to calculate $\overline{KB}$ at a series of drafts; plot $\overline{KB}$.
6. Calculate plot $\overline{KM}$ from $\overline{KB}$ and $\overline{BM}$, (4) and (5), at each draft.
7. Calculate $\overline{GM}_L$ using assumed $\overline{KG}$, $\overline{KB}$ from (5), $\overline{BM}_L$ from (4); using ship length between draft marks, calculate, plot *MTcm* at each draft.
8. Using *TPcm, LCF* (3), and ship length between draft marks, calculate coming plot increase in displ. for 1 cm trim by stern at each draft.
9. Read displacement from curve (2), calculate, plot C_B; calculate, plot C_M from midship Bonjean curve; calculate, plot C_P from C_M and C_B at each draft.
10. Read waterline half breadths at appropriate stations and drafts, calculate wetted surface over main body of ship at each draft using differential surface area method (see Section 7).
11. Girth stations at ends of ship, approximate wetted surface of ends; calculate, plot wetted surface of complete ship by adding wetted surface of main body from (10) at each draft.
12. Using waterplane areas from (3), integrate vertically to get displacement and $\overline{KB}$, check results obtained in (2) and (5).

Fig. 29 Sequence of Curves of Form calculations

5.15 Summary of Calculations. A concise summary of results should be furnished with the curves of form calculations to assist in plotting the curves and serve as a permanent record. Values should be listed at all drafts at which calculations are performed. Drafts should be spaced closely enough to allow drawing the curves without ambiguity. Table 10 is a condensed summarization of the plotted values in Fig. 23 for the example ship.

For most ship-shaped forms, the plotted curves will be fair, with few points of inflection. The presence of any unfairness usually indicates an error in the calculations—as from incorrect input data—or an error in plotting.

All of the data given in Table 10 apply to the molded form, with the exception of total displacement in fresh and salt water, and wetted surface, described in Section 7. Ordinarily, the other data, based on the molded form, are used without correcting for the effect of appendages, because such effects are so small as to be negligible as far as practical purposes are concerned—except in some cases for LCB. (Section 5.7).

Since final curves of form are for the benefit of operating personnel, the curve sheet furnished the ship owner should show drafts to the bottom of the keel, with the distance from the molded base line to the bottom of keel clearly noted.

5.16 Computer Applications. Calculations for curves of form require numerous repetitive calculations which were formerly performed on desk calculators. Consequently, one of the earliest applications of digital computers in naval architecture was in the calculation of hydrostatic properties. According to Lasky and Daidola (1977), in a survey of computer utilization by more than 200 ship design offices, the largest area of design activity to which computers are applied is hydrostatics, with about 80 percent of the responders so involved. There are known to be a large number of programs in existence for such hydrostatic calculations, and the already widespread application of such programs is expected to grow.

The basic input must be the overall dimensions of the ship and the offsets at a series of waterlines and stations together with a definition of the end profiles, stated or implied. It is a matter of judgment as to how many waterlines and body plan stations are needed to assure valid hydrostatic calculations. Clearly more closely spaced waterlines are needed when the vessel's form changes rapidly with draft, as normally applies at the lowest drafts, but the spacing should be appropriate to the rule of integration adopted. Figure 29 shows a possible sequential scheme for performing the various calculations. Fig. 30 shows a typical computer-generated body plan derived from stored offsets.

Some programs perform many of the hydrostatic calculations needed for intact and damage stability studies described in Chapters II and III, in addition to curves of form, inasmuch as the hull surface offsets, once defined by the computer, can serve as common input data. When the computer is linked to a cathode ray scope and/or a plotter, the results may be displayed for checking and/or hard copy curves can be

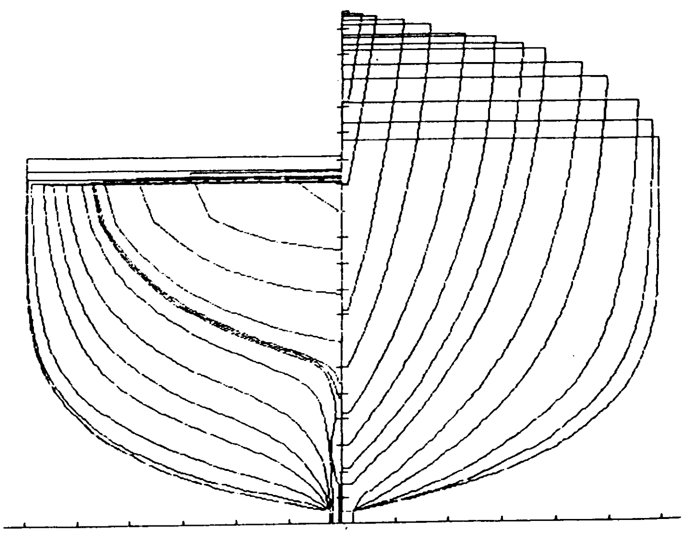


Fig. 30 Computer printout of body plan of small vessel in Fig. 31

plotted, in addition to appearing in the computer output in tabular form.

The U.S. Navy's Ship Hull Characteristics Program (SHCP) NAVSEA (1976) incorporates these features and is widely used in U.S. design offices and shipyards for both commercial and naval ships. It comprises a set of sub-programs which perform any or all of the following naval architectural calculations:

- Curves of form and Bonjean Curves.
- Longitudinal shear and bending moment (still water and in wave).
- Trim lines after flooding.
- Floodable length.
- Limiting drafts for survival after flooding.
- Curves of intact static stability and cross curves.
- Curves of static stability and cross curves, damaged.

Common input for the above are the hull offsets, which may be read from the lines drawing and entered by means of punched cards. Or a digitizer may be used to trace the body plan, read the points and enter them in the computer. A more economical procedure is to enter the offsets directly from the stored HULDEF data (Section 1.15).

When utilizing this program the user is not obliged to use the station or waterline location and spacing for which the lines have been drawn; rather, an odd number (minimum 3, maximum 41) of body plan stations are chosen. Each station must have a non-zero sectional area when fully immersed. Section offsets are specified at between 2 and 29 points for each body plan station which is assumed describable by a series of 2nd order curve segments. These segments are consistently taken between odd numbered waterlines, for integration by Simpson's First Rule.

From the offsets the Ship Data Table (SDT) is set up, which is the common data base for all of the sub-programs. It contains the following, calculated by Simpson's First Rule, for each point (waterline) on each section:

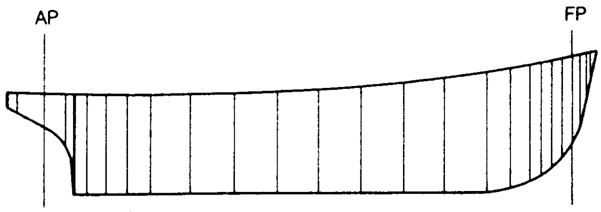
(a) Half-breadth and height above baseline; and cumulative properties (above baseline).

(b) Full section area (both sides).

(c) Transverse centroid of half section.

(d) Vertical centroid of section.

(e) Half-girth.



LBP OF SHIP = 12.5 STATIONS = 300 FT.
STATION SPACING = 24 FT.
STATION DISTANCE FROM F.P.
= STATION NO.x STATION SPACING.
STATIONS SHOWN:

−0.500	2.000	11.000
−0.417	3.000	11.500
−0.333	4.000	11.742
−0.167	5.000	11.771
0.000 (FP)	6.000	11.800
0.250	7.000	12.000
0.500	8.000	12.500 (AP)
0.750	9.000	13.000
1.000	10.000	13.417
1.500	10.500	

Fig. 31 Typical station spacing for integration by SHCP

The SDT can be printed out and/or retained in the computer memory.

In subsequent calculations Simpson's First Rule is also used between odd-numbered stations for longitudinal integration. In order to handle extremities of curves which do not terminate at selected stations, the program itself extrapolates the curve to the axis and determines one offset midway between the extremity so found and the first known offset.

The first subprogram produces the data for the usual curves of form for up to 21 waterlines and 7 trims. Calculated properties are presented in tabular form, and optional plots are provided of curves of form, waterlines and sectional areas (Bonjean curves).

Fig. 31 shows a typical spacing of stations for a small vessel integrated using the SHCP. Table 11 shows part of the output, in English units as customarily used by the U.S. Navy.

If computer output is to serve as the final record for use by ship's personnel, it is important that the data be presented in a completely clear way, bearing in mind the environment in which they will be used.

Table 11—Typical Output from Computer Calculation of Ship Hydrostatics using SHCP Program

SHIP- SHCP SAMPLE SHIP S.S.SUSAN GAIL SERIAL NUMBER- 717 DATE- 5/25/84

HYDROSTATICS - PART I TRIM 0.000 FEET

	DRAFT	VOLUME	DISPLACEMENT	LCB	KB	WETTED SURFACE	PRISMATIC COEF	WPLANE COEF	WPLANE I COEF
	2.00	1722.	49.2	-9.56	1.03	2143.	0.759	0.590	0.372
	10.00	51593.	1474.1	8.37	6.81	12751.	0.547	0.580	0.410
	18.00	161991.	4628.3	6.41	11.90	20253.	0.599	0.666	0.502
	26.00	310907.	8883.1	4.30	16.81	26338.	0.644	0.738	0.578
	30.00	395435.	11298.2	3.22	19.20	29195.	0.665	0.770	0.607
DWL	36.00	531140.	15175.4	1.57	22.74	33410.	0.693	0.815	0.651
	44.00	725939.	20741.1	-0.80	27.39	39223.	0.729	0.875	0.707

HYDROSTATICS - PART II TRIM 0.000 FEET

	DRAFT	WPLANE AREA	LCF	TPI	CIDOFTS	LONG. BM	TRNSV BM	LONG. KM	TRNSV KM	MT1
	2.00	1271.	0.12	3.03	-0.01	2965.5	2.00	2966.6	3.04	40.5
	10.00	10638.	7.38	25.33	-7.47	686.8	45.48	693.6	52.29	281.2
	18.00	16552.	3.75	39.41	-5.91	415.5	44.05	427.4	55.95	534.2
	26.00	20402.	0.31	48.58	-0.60	313.8	36.41	330.6	53.22	774.2
	30.00	21791.	-1.80	51.88	3.74	284.9	32.21	304.1	51.41	894.1
DWL	36.00	23401.	-4.77	55.72	10.64	251.9	26.84	274.6	49.58	1061.7
	44.00	25202.	-9.58	60.00	22.99	225.4	21.58	252.8	48.97	1298.9

SECTIONAL AREAS IN SQUARE FEET - PART 1 TRIM 0.000 FEET

	STATION / DRAFT	-0.500	-0.417	-0.333	-0.167	0.000	0.250	0.500	0.750
	2.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.28
	18.0	0.00	0.00	0.00	0.00	0.00	3.26	38.06	99.29
	26.0	0.00	0.00	0.00	0.00	5.25	69.01	164.73	289.98
	30.0	0.00	0.00	0.00	0.00	26.25	126.51	256.31	414.52
DWL	36.0	0.00	0.00	0.00	8.56	81.38	238.81	423.31	631.97
	44.0	0.00	0.00	0.00	55.55	190.27	431.89	691.69	967.61

SECTIONAL AREAS IN SQUARE FEET - PART 2 TRIM 0.000 FEET

	STATION / DRAFT	1.000	1.500	2.000	3.000	4.000	5.000	6.000	7.000
	2.0	0.00	0.44	7.02	7.20	7.37	7.48	7.64	7.48
	10.0	26.06	90.64	144.83	217.50	266.63	293.07	309.75	308.99
	18.0	169.04	325.90	457.47	649.16	781.84	857.61	895.07	894.64
	26.0	411.40	670.16	868.00	1211.42	1426.43	1547.90	1598.99	1600.82
	30.0	563.29	873.90	1134.03	1522.66	1775.34	1916.85	1972.57	1973.82
DWL	36.0	822.79	1203.40	1531.72	2009.91	2314.97	2481.36	2542.69	2544.57
	44.0	1215.08	1699.35	2161.01	2686.58	3050.62	3242.45	3310.07	3311.95

Table 11—Typical Output from Computer Calculation of Ship Hydrostatics using SHCP Program

SHIP- SHCP SAMPLE SHIP S.S.SUSAN GAIL SERIAL NUMBER- 717 DATE- 5/25/84

HYDROSTATICS - PART I TRIM 0.000 FEET

DRAFT	VOLUME	DISPLACEMENT	LCB	KB	WETTED SURFACE	PRISMATIC COEF	WPLANE COEF	WPLANE I COEF
2.00	1722.	49.2	-9.56	1.03	2143.	0.759	0.590	0.372
10.00	51593.	1474.1	8.37	6.81	12751.	0.547	0.580	0.410
18.00	161991.	4628.3	6.41	11.90	20253.	0.599	0.666	0.502
26.00	310907.	8883.1	4.30	16.81	26338.	0.644	0.738	0.578
30.00	395435.	11298.2	3.22	19.20	29195.	0.665	0.770	0.607
DWL 36.00	531140.	15175.4	1.57	22.74	33410.	0.693	0.815	0.651
44.00	725939.	20741.1	-0.80	27.39	39223.	0.729	0.875	0.707

HYDROSTATICS - PART II TRIM 0.000 FEET

DRAFT	WPLANE AREA	LCF	TPI	CIDOFTS	LONG. BM	TRNSV BM	LONG. KM	TRNSV KM	MT1
2.00	1271.	0.12	3.03	-0.01	2965.5	2.00	2966.6	3.04	40.5
10.00	10638.	7.38	25.33	-7.47	686.8	45.48	693.6	52.29	281.2
18.00	16552.	3.75	39.41	-5.91	415.5	44.05	427.4	55.95	534.2
26.00	20402.	0.31	48.58	-0.60	313.8	36.41	330.6	53.22	774.2
30.00	21791.	-1.80	51.88	3.74	284.9	32.21	304.1	51.41	894.1
DWL 36.00	23401.	-4.77	55.72	10.64	251.9	26.84	274.6	49.58	1061.7
44.00	25202.	-9.58	60.00	22.99	225.4	21.58	252.8	48.97	1298.9

SECTIONAL AREAS IN SQUARE FEET - PART 1 TRIM 0.000 FEET

STATION / DRAFT	-0.500	-0.417	-0.333	-0.167	0.000	0.250	0.500	0.750
2.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.28
18.0	0.00	0.00	0.00	0.00	0.00	3.26	38.06	99.29
26.0	0.00	0.00	0.00	0.00	5.25	69.01	164.73	289.98
30.0	0.00	0.00	0.00	0.00	26.25	126.51	256.31	414.52
DWL 36.0	0.00	0.00	0.00	8.56	81.38	238.81	423.31	631.97
44.0	0.00	0.00	0.00	55.55	190.27	431.89	691.69	967.61

SECTIONAL AREAS IN SQUARE FEET - PART 2 TRIM 0.000 FEET

STATION / DRAFT	1.000	1.500	2.000	3.000	4.000	5.000	6.000	7.000
2.0	0.00	0.44	7.02	7.20	7.37	7.48	7.64	7.48
10.0	26.06	90.64	144.83	217.50	266.63	293.07	309.75	308.99
18.0	169.04	325.90	457.47	649.16	781.84	857.61	895.07	894.64
26.0	411.40	670.16	868.00	1211.42	1426.43	1547.90	1598.99	1600.82
30.0	563.29	873.90	1134.03	1522.66	1775.34	1916.85	1972.57	1973.82
DWL 36.0	822.79	1209.40	1531.72	2009.91	2314.97	2481.36	2542.69	2544.57
44.0	1215.08	1699.35	2161.01	2686.58	3050.62	3242.45	3310.07	3311.95

Section 6

Bonjean Curves

6.1 Curves of Areas of Transverse Sections. Fig. 32(a) shows a typical transverse section through a ship on one side of the centerline, such as a body plan station. The area $KCL_1 W_1 K$ from the baseline up to waterline $W_1 L_1$ may be obtained by one of the rules of integration, or by planimeter or integrator. Twice the area, plotted to a convenient scale and at the same draft as W_1L_1, would appear as the point Q in Fig. 32 (b). Similarly, the area $KCLWK$ from the baseline up to WL could be obtained and would give the point P in Fig. 32(b). The curve $K'QPF'T$, Fig. 32 (b) thus represents the area of the full section on both sides of the centerline, from the baseline up to any waterline.

For wooden vessels, the half section should be taken to the outside of the planking, but for steel vessels, it is customary to draw the curve of areas for sections taken to the molded line.

In cases where vessels have unusually large appendages, it may be desirable to construct the curve of transverse section area with the inclusion of the shell thickness, corrected for the obliquity of the vessel's form, together with the cross sectional area of other appendages such as bilge keels. A longitudinal integration of such total cross section areas, together with the volume of appendages not intersected by the sections, would give the total displacement of the ship, but the calculation of the curves of cross sectional area would be too laborious for general use.

The curves of cross sectional area for all body plan stations are collectively called *Bonjean Curves*.[5] One of the principal uses of Bonjean Curves is determining volume of displacement of the ship at any level or trimmed waterline.

A convenient way to calculate Bonjean Curves is by the use of closely spaced waterlines at the lower levels and the use of the 5, 8, −1 rule described in Section 4.6. Table 12 shows the calculation for one point on a Bonjean Curve for the example ship. In the event the lowest contour of the station is considered to curve too sharply for a satisfactory parabolic approximation using available halfbreadths, a planimeter may be used. Further points on the Bonjean Curves may then be found, with the area between the next pair of waterlines added to that below. The moment of area of each section about the molded baseline may be found by an adaptation of the 3, 10, −1 rule. If these vertical moments be integrated longitudinally, one may find the moment of volume of displacement about the molded baseline, and hence $\overline{KB}$, the vertical height of center of buoyancy. The vertical moments of sections across the vessel up to any waterline may be useful in problems which arise in the case of a ship flooded throughout part of its length (Chapter III).

6.2 Construction of Bonjean Curves. Bonjean Curves may be plotted in either of two ways. Fig. 33 shows the curves for the ship shown in Fig. 1 plotted against a common scale of draft, with the cross sectional areas for stations in the forebody and amidships plotted to the right of the vertical axis and those for the afterbody plotted to the left. The draft scale may represent keel drafts, or molded drafts, but the distance from the molded baseline to the bottom of keel should be shown. Such a presentation has the advantage of compactness, and uses one scale of cross sectional area. It is convenient to show a contracted profile of the ship adjoining the curves.

An alternative plot is that shown by Fig. 34 in which a separate horizontal scale of cross sectional area is provided for each curve, and the curves are superposed on a contracted profile of the ship; in the latter case, the vertical axes coincide with the associated station lines in the profile. This arrangement is convenient for placing and locating trim lines on the profile, but has the disadvantage that the horizontal area scales for each station may be difficult to distinguish, one from the other, at areas of overlap. Draft scales corresponding to those on the ship should be shown at the appropriate locations on the profile.

[5] Named after a French naval engineer of the early nineteenth century.

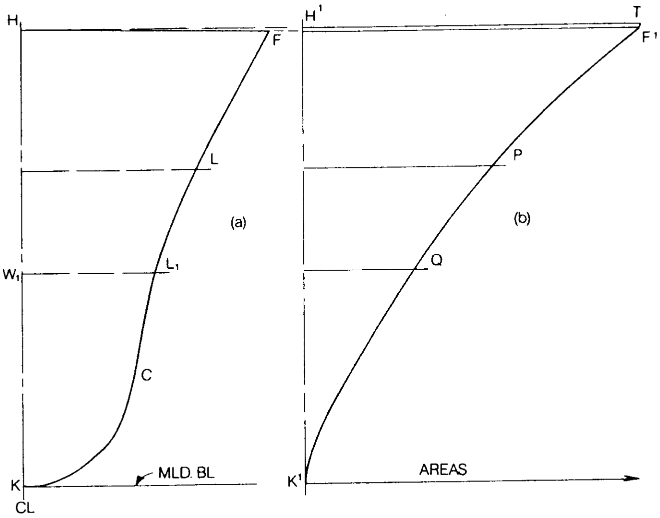


Fig. 32 Body plan section (a) and Bonjean Curve (b)

6.3 Uses of Bonjean Curves. As noted in Section 5.7, a standard method of calculating volume of displacement and LCB is by integrating transverse sectional areas. If the waterline at which the ship is floating is not for the even keel condition, Bonjean Curves are particularly useful. In the case of a trimmed waterline, the trim line may be drawn on the profile of the ship and drafts read at which the Bonjean Curves are to be entered. By drawing a straight line across the contracted profile of Figs. 33 and 34, the drafts at which the curves are to be read appear directly at each station.

Inasmuch as the curves of form are constructed for the ship in the even keel condition and most ships are not wall-sided, accurate hydrostatic characteristics for cases with a significant degree of trim are not in general obtainable from the curves of form and one must

Table 12—Calculation of Point on Bonjean Curve at Lower Level

(For Station 8 on Example Ship at 1m waterline)

Height above Baseline m	Halfbreadth m	Multiplier	Product
0	0.66	5	3.30
1	3.08	8	24.64
2	4.02	−1	−4.02
			$\Sigma = \overline{23.92}$

Transverse section area below 1m waterline

$$= \frac{s}{12} \times \Sigma = \frac{1}{12} \times 23.92$$

$$= 1.99 \text{ m}^2\text{, one side of ship,}$$

$$= 3.99 \text{ m}^2\text{, both sides of ship.}$$

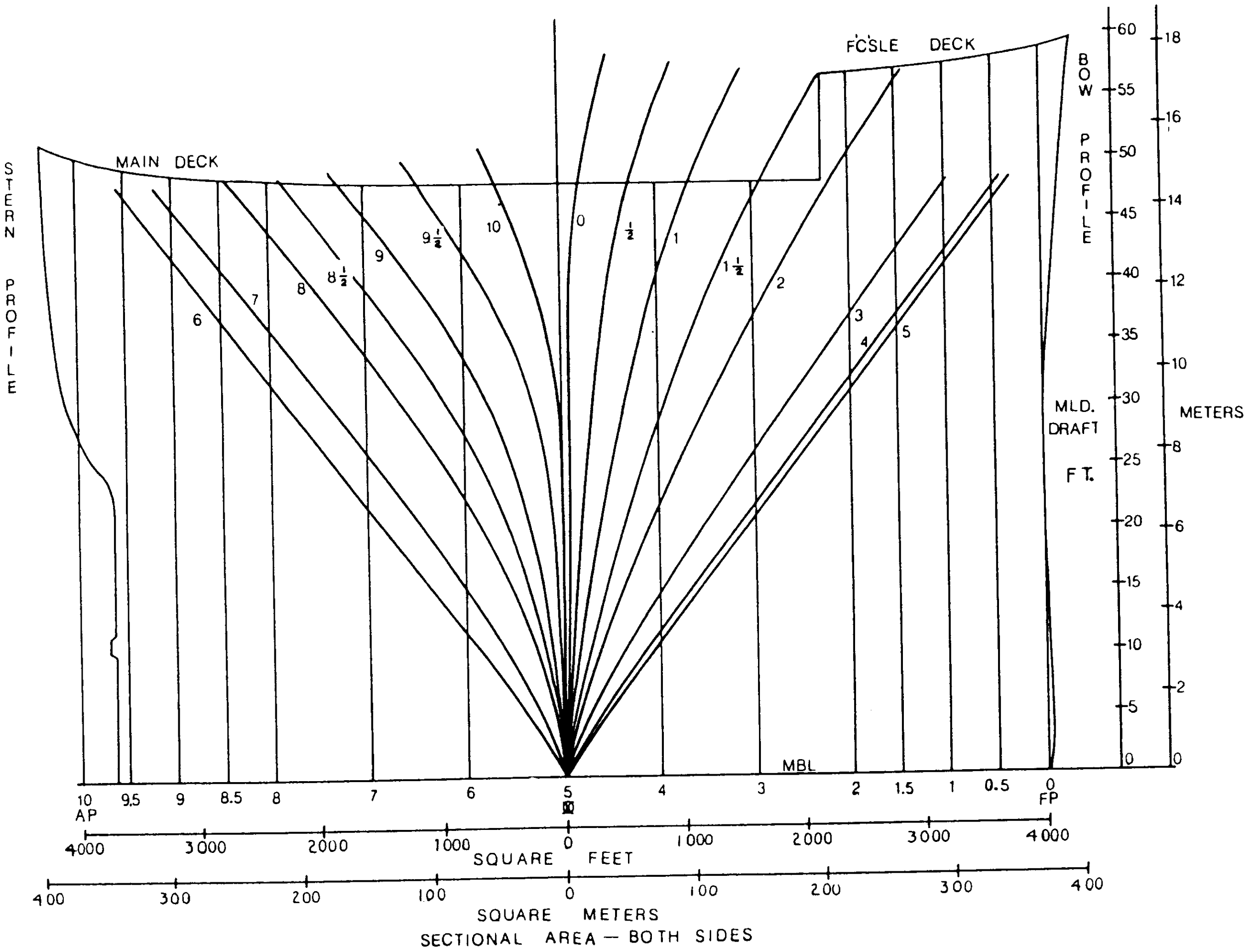


Fig. 33 Bonjean Curves—common vertical axis

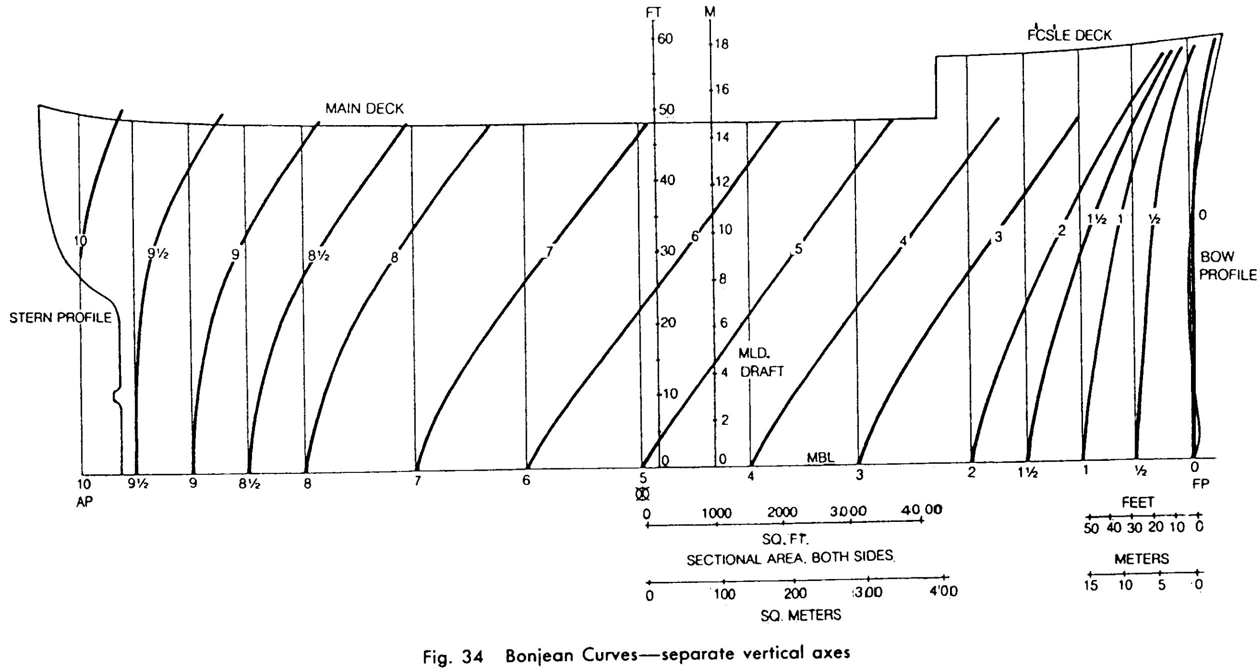


Fig. 34 Bonjean Curves—separate vertical axes

perform a complete longitudinal integration at the trimmed waterline (trim line) under consideration. The Bonjean Curves provide the basic input for such calculations. Cases where this is needed may occur in connection with launching, discussed in *Ship Design and Construction* (Taggart, 1980), and when the end compartments of a ship are flooded, as discussed in Chapter III. If the ship be considered in the crest or trough of a wave of known profile, as may be assumed for longitudinal strength calculations, again the displacement of the ship can be calculated, as discussed in Chapter IV.

Section 7
Wetted Surface

7.1 Definitions and Uses of Wetted Surface. For a vessel floating at a given waterline, the total area of its outer surface in contact with the surrounding water is known as its *wetted surface.* When estimating the frictional resistance to the motion of a vessel through the water, it is important to know the vessel's total wetted surface up to any waterline at which the vessel may operate. The subject of frictional resistance, and corrections to the wetted surface for the ship's wave profile, is treated in Chapter V.

The wetted surface may be used in estimating the amount of paint required to coat the vessel's bottom up to a given waterline. Also, the wetted surface below the waterline may be added to the area of the topsides above the waterline to obtain the total area of the shell plating. Thus, the approximate weight of the shell may be estimated as well as the paint required for it.

Wetted surface is customarily calculated at various waterlines for a new ship and appears as one of the curves of form. In some cases additions are made for appendages, such as stem, stern frame, rudder, propeller shaft bossing and bilge keels.

7.2 Calculation of Wetted Surface. The underwater surface of the molded form is the principal component of the total wetted surface of a ship. In a fine, high-speed, multiple-screw ship it may amount to 85 percent of the total wetted surface. In a full single-screw ship it may amount to 99 percent of the total. The wetted surface of the shell plating is virtually that of the molded form; therefore, calculations of the molded surface of the hull plus that of appendages extending beyond the shell may be considered the entire wetted surface.

The wetted surface of the molded form may be obtained by calculating various portions directly from the lines drawing, and estimating other portions closely. The calculation method has traditionally been that of drawing an expansion of the molded surface up to the desired waterline, and measuring the area enclosed by the expansion, it being assumed the area of the expansion is virtually that of the molded surface (See next section.).

At each transverse section of the body plan, the distance along the contour of the section from the centerline at the bottom up to any given waterline is known as the half-girth of the section up to that waterline. The half girth may be obtained by bending a thin flexible batten around the section, or a straight measuring scale or strip of paper may be placed in contact with the curve of the section at the starting point and thereafter kept in contact with and tangent to the curve at successive points, by rotating the strip of paper slightly with the paper held in place at the point of contact by the point of a pencil. The measuring scale or strip of paper should be rotated continually about its successive points of contact along the curve. Alternatively, a map measurer may be rolled along the section perimeter, following the curve carefully and noting the revolutions of the wheel; these may be interpreted, by reference to a calibrated scale, as girthed distance.

The half-girths of the various sections may be plotted as ordinates on their respective stations along a base line representing the length of the vessel. A fair curve passed through such points will enclose an area known as the transverse expansion of the molded surface of one side of the vessel up to the given waterline. Similarly, the transverse expansion of one side of the vessel's surface may be obtained between any two waterlines, or up to any deck line. Fig. 35 shows these transverse expansions up to several successive waterlines, constructed from the lines drawing, Fig. 1.

The area of the transverse expansion of a vessel's molded surface up to a given waterline may be calculated readily and considered as a first approximation to the true area of the vessel's molded wetted surface. This approximation is usually correct to within about 2 percent and for many purposes this degree of accuracy will suffice.

7.3 Graphical Corrections to Wetted Surface From Transverse Expansion. The molded surface is usually composed, in large part, of surfaces of compound curvature and so cannot be expanded into a plane. Hence, the area enclosed by the transverse expansion does not properly account for the obliquity of the vessel's lines. To overcome this inaccuracy, several alternative graphical corrections have been developed. Fig. 36 shows a portion of a half breadth view in the forebody

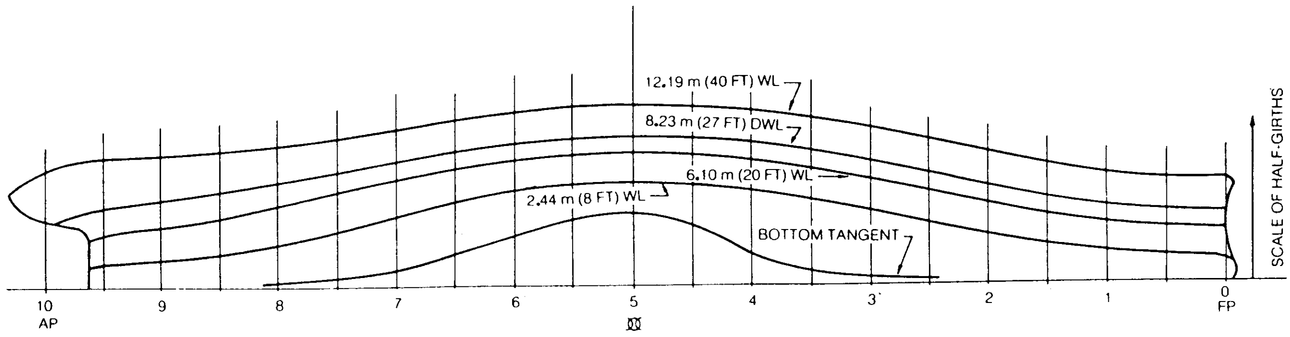


Fig. 35 Transverse expansion of half girths

of a ship, with the expansion of half girths shown above it. The 5 m waterline may be considered as giving the average slope of the 4 m and 6 m waterlines. By girthing the 5 m waterline, starting at amidships, station 5, and working forward, using either a thin, flexible batten or a strip of paper, the distances along the true shape of the waterline at which stations are crossed may be found. When these distances are laid out along the baseline of the expansion, it will be found that point 1 moves to point 1′, point ½ moves to point ½′, while the bow contour moves forward the maximum amount. The area under the dashed curve is then a closer approximation to the molded wetted surface of the vessel between the 4 m and 6 m waterlines than is the area under the uncorrected transverse expansion. This procedure is known as rectification of waterlines.

Another method, which is numerically equivalent to that of rectification of waterlines applies a secant correction to the half girths. Thus, in Fig. 36, the straight line *CD*, connecting the 5 m waterline half breadths at station 1½ and 2½, has a waterline angle ϕ which is practically the average angle of the average waterlines between the 4 m and 6 m waterlines, and in way of station 2. Thus, the length of the molded surface between these waterlines, and between stations 1½ and 2½ is quite close to *CD*. However, $CD = s \cdot \sec \phi$ where s is full station spacing. Thus, if the half girth at Station 2 be multiplied by sec ϕ, the area enclosed by the stations and line *GHF* in Fig. 36 will be a closer approximation to the true wetted surface than the area enclosed by the stations and line *KTN*. The secant method of modifying half-girths is probably more convenient for calculation purposes than is rectification of waterlines, inasmuch as station ordinate locations are not changed.

7.4 Integration of Differential Surface Areas. The foregoing methods are inherently approximate. The area of a transverse expansion is slightly less than the true surface area, while the rectification of waterline/secant half-girth correction method over compensates for the effect of obliquity on the transverse expansion, giving an area slightly too large. A more direct method of finding wetted surface is to integrate the differential surface area along the ship.

The elementary area of a curved three-dimensional surface, defined with respect to mutually perpendicular x, y, z axes, may be found as the product of elementary area in the x, z plane δx, δz and the secant of inclination of the surface from the x, z plane.

In vector analysis it is shown that for a surface $y = f(x,z)$, the angle β between a vector normal to the surface and the y axis is given by,

$$\cos \beta = \frac{1}{\sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2}}.$$

Then

$$\sec \beta = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2}.$$

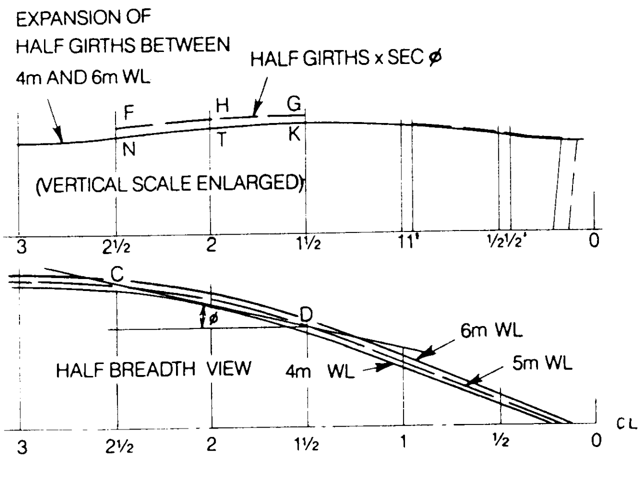


Fig. 36 Correction to transverse expansions for obliquity

Assume x distances are longitudinal distances along the ship, y distances are halfbreadths to the molded surface, and z distances are heights above the molded baseplane to the molded surface.

Consider the unit differential area δx, δz in Fig. 37 in the longitudinal centerplane of the vessel and project this to the molded surface, giving differential surface area δs.

Then

$$\delta s = \delta x \cdot \delta z \sqrt{1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2}, \text{ or area,}$$

$$S = \sum\sum \delta x\, \delta z \sqrt{1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2}, \text{ or} \qquad (50)$$

$$S = \iint \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dx\, dz.$$

There seems to be no simple way of reducing this double integral to an integral in one variable, to permit calculating surface area by a single step rule of integration. We are obliged instead to find the area incrementally, the simplest and most useful way for a ship's molded surface being to calculate the surface between pairs of waterlines which are reasonably closely spaced. That is, let δz be a constant difference in draft and then integrate longitudinally using a rule of integration, such as Simpson's First or Second Rule to find the wetted surface between each pair of waterlines.

Table 13 illustrates a tabular calculation for finding wetted surface based upon differential surface area for the area between two waterlines of a tanker, and between stations 15 and 19 (20 stations LBP). The calculation makes use of Simpson's Second Rule as the primary rule of integration, together with Simpson's First Rule. It will be seen that the vertical station shape slope $\frac{\delta y}{\delta z}$ is obtained from the difference in station half-breadths above and below the area to be integrated, whereas the longitudinal waterline slope is obtained as the mean of differences between waterline half-breadths forward of and abaft the station in question.

The calculation routine avoids the necessity for measuring half girths but makes maximum use of the molded surface offsets. As such, it is easily adapted to programming on a digital computer.

7.5 Wetted Surface Coefficients. In order to compare the wetted surface of different ships it is useful to calculate a dimensionless coefficient which relates the wetted surface to the basic characteristics of the hull. Such a coefficient is C_{WS}, where,

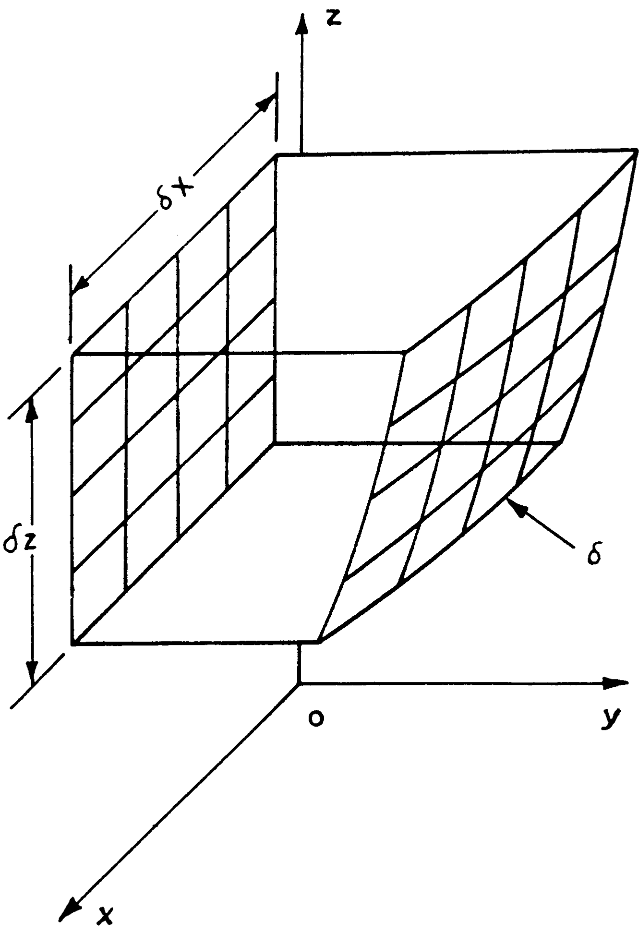


Fig. 37 Differential surface area

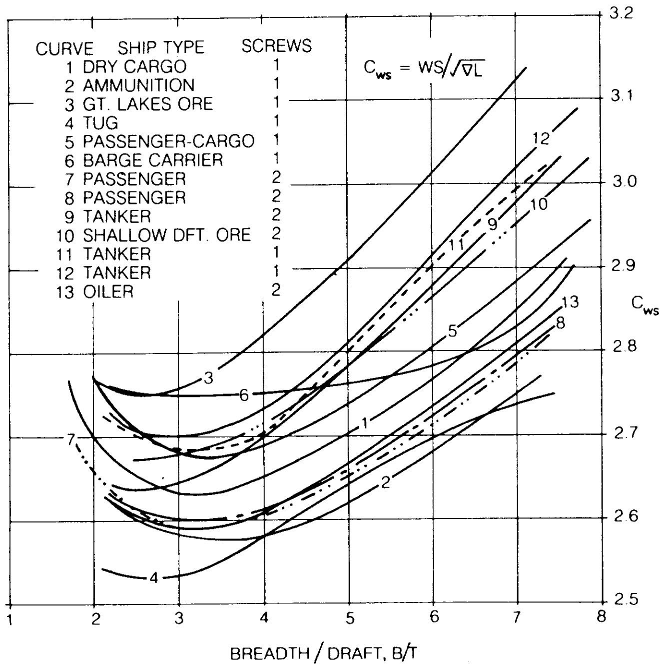


Fig. 38 Wetted surface coefficients

Table 13—Calculation of Wetted Surface of Portion of Tanker by Differential Surface Area Method
(Between Stations 15 and 19 and between the 3 m and 6 m waterlines)

Sta	HB 6 m	HB 3 m	$\left(\frac{\delta y}{\delta z}\right)$	$\left(\frac{\delta y}{\delta z}\right)^2$	Next sta ford	HB 6 m	HB 3 m	Next sta aft	HB 6 m	HB 3 m	Mean $\left(\frac{\delta y}{\delta x}\right)$	$\left(\frac{\delta y}{\delta x}\right)^2$	Sum	$[\text{Sum}]^{1/2}$	SM	Product
15	19.66	18.41	0.42	0.17	14	20.12	19.84	16	17.56	15.56	−0.12	0.01	1.18	1.09	1	1.09
16	17.56	15.97	0.70	0.49	15	19.66	18.41	17	13.38	11.16	−0.24	0.06	1.55	1.24	3	3.72
17	13.38	11.16	0.74	0.55	16	17.56	15.47	18	8.14	6.64	−0.33	0.11	1.66	1.29	3	3.87
18	8.14	6.64	0.50	0.25	17	13.38	11.16	19	2.62	2.16	−0.35	0.13	1.38	1.17	1.444	1.69
18½	5.43	4.39	0.35	0.12	18	8.14	6.64	19	2.62	2.16	−0.36	0.13	1.25	1.12	1.778	1.99
19	2.62	2.16	0.15	0.02	18½	5.43	4.39	19½	−0.22*	−0.28*	−0.37	0.14	1.16	1.08	0.444	0.48
															$\Sigma =$	12.84

HB's are waterline halfbreadths

$\delta z = (6 - 3) = 3\text{m}$

δx = station spacing = 13.94 m.

$$\text{Mean}\left(\frac{\delta y}{\delta x}\right) = \left(\frac{HB_{A6} - HB_{F6} + HB_{A3} - HB_{F3}}{4 \cdot \delta x}\right),$$

where $4 \cdot \delta x = 4 \times 13.94 = 55.76\text{m}$ in way of stations,
$4 \cdot \delta x = 2 \times 13.94 = 27.88\text{m}$ in way of half stations.

$$\text{Sum} = 1 + \left(\frac{\delta y}{\delta z}\right)^2 + \left(\frac{\delta y}{\delta x}\right)^2$$

* Negative halfbreadths from extrapolating waterline to station.

Integrating factor, IF $= 2 \cdot \frac{3}{8} \cdot \delta x \cdot \delta z = 2 \cdot \frac{3}{8} \cdot 13.94 \cdot 3 = 31.635$. HB_A is halfbreadth aft

Wetted surface, both sides $= \text{IF} \cdot \Sigma = 31.365 \cdot 12.84 = 402.7\text{m}^2$. HB_F is halfbreadth forward

$$C_{WS} = WS/\sqrt{\nabla \cdot L} \quad (49)$$

Here WS is wetted surface up to any waterline,
∇ is volume of displacement at that waterline,
L is length of vessel.

Values of C_{WS} range between about 2.6 and 2.9 for usual ships of normal form according to plots in Saunders (1957). There is a noticeable dependence of C_{WX} on B/T, beam to draft ratio, and on C_M, midship coefficient. According to Saunders' plots, minimum C_{WS} occurs approximately with $B/T = 2.9$ and $C_M = 0.92$.

Fig. 38 plots wetted surface coefficient against B/T for a number of unrelated ships, with the ship type and number of propellers noted. The coefficients have been developed from wetted surface curves on the curves of form sheet for each vessel. In all cases C_{WS} reaches a minimum in the range of B/T from 2.5 to 3.75, with C_{WS} increasing for B/T beyond the minimum point. The increase of C_{WS} for low B/T is believed to reflect the greater submergence of the stern overhang which tends to accompany draft increases beyond the design draft.

Wetted surface may also be estimated by reference to data on published hull form series such as Series 60 (Todd, et al, 1957).

Unusual ships may be expected to have wetted surface coefficients substantially different from those of Fig. 38, or those shown by Saunders. In general it may be expected that C_{WS} will increase because of hard sections, chines, knuckles and unusually large appendages. The inclusion of the wetted surface of stern transoms, internal wells or the mating surfaces of an integrated tug-barge in estimating frictional resistance may not be appropriate, as discussed in Chapter V.

Section 8
Capacity

8.1 General. A basic characteristic of any ship is the size of the load that it is able to carry. Thus, two fundamental questions arise: (a) What is the volume of space available for cargo—or cargo *capacity*? (b) What is the weight of cargo that can be carried at full load draft—or cargo *deadweight*?

Under considerations of capacity are included the volume of all cargo spaces, store rooms and tanks and the location, vertically, longitudinally, and transversely of the centroid of each such space to allow finding the weight (and center of gravity) of the variable weights, or deadweight of the ship. This information is needed to check the adequacy of the vessel's size, and to determine its trim and stability characteristics. The calculations are called capacity calculations and lead to capacity curves and plans.

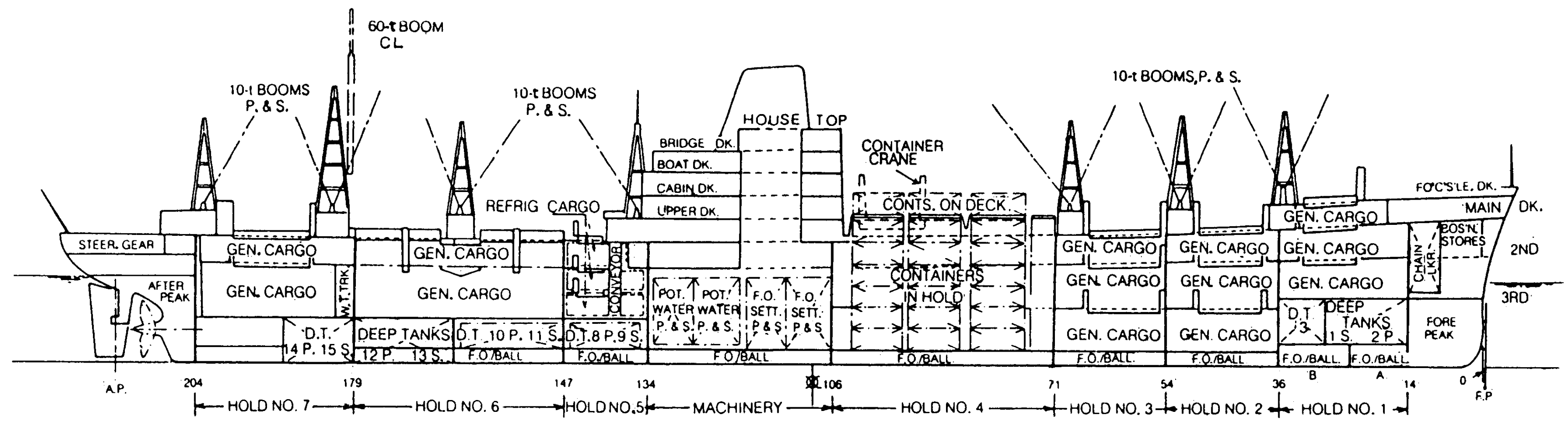
The total deadweight of a merchant ship is the difference between the full-load displacement weight and the light ship weight—the latter consisting of the weight of hull steel, machinery (wet), and outfit. The actual "payload" or cargo deadweight is obtained by deducting the typical maximum values of the variable weights of fuel, stores, fresh water, water (or other removable) ballast, crew and their effects from the total deadweight.

In this book we must use the term weight loosely, for when using SI units of kilograms or metric tons we are really speaking about mass. However, if inch-pound units are retained—lbs and long tons—we are then speaking correctly of weight. Similarly, deadweight and displacement (as explained in section 2.2) can be in either SI mass units or in inch-pound weight units.

8.2 Capacity Plan. Fig. 39 shows, in abbreviated form, the capacity plan for a multipurpose dry cargo ship. Such a plan is prepared for the use of the ship owner, and summarizes in convenient form the amount of cargo, fuel, fresh water and stores which the ship may carry, and the spaces into which these will go. The amount of elaboration on the actual plan varies at different shipyards, and depends upon owners' requirements. There is always an outline inboard profile showing the location of tanks, store and cargo spaces and frequently there are also deck plans showing the arrangement of these spaces, as well as sectional views at various frame locations along the ship. The plan includes the principal dimensions of the ship, and shows, usually in tabular form, the name, location and volume of each cargo space, tank, consumable stores space, etc., as well as its longitudinal and vertical centroid when filled, and its transverse centroid, if the space is unsymmetrically disposed about the vessel's centerline. If large units of cargo are to be carried, such as shipping containers or cargo barges, the location of the various vertical tiers and longitudinal rows are shown, as well as how the units are positioned transversely. In the event that the ship is intended to carry specific amounts of deck cargo, such as deck-stowed containers, these also are shown.

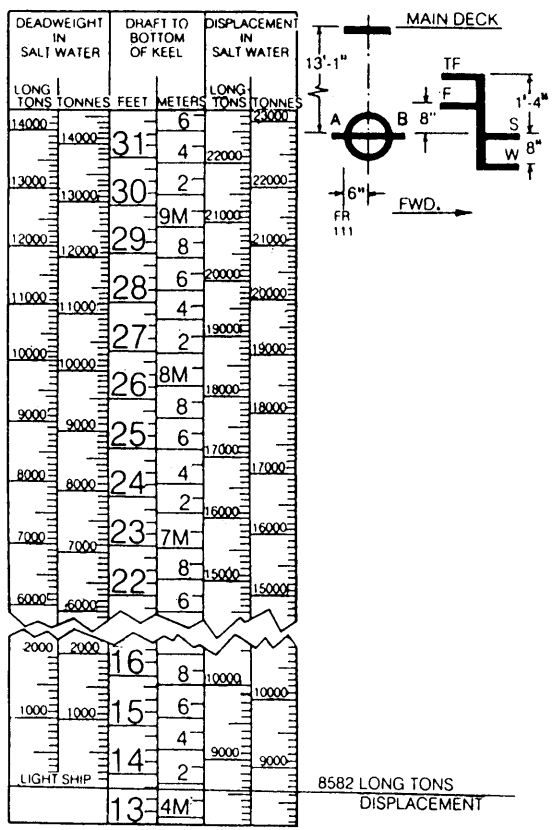
The plan includes a displacement scale alongside a scale of drafts, and draft markings as they appear on the side at the ship at amidships, for all drafts from the light condition to full load. Also usually shown next to the draft scale are $TPcm$ and $MTcm$. Freeboard

(Continued on page 54)



PRINCIPAL PARTICULARS

LENGTH OVERALL	171.8 m (563'-7¾")
LENGTH BETW. PERPS. (ABS)	161.1 m (528'-6")
BREADTH, MOLDED	23.2 m (76'-0")
DEPTH, MLD., MAIN DK AT SIDE	13.6 m (44'-6")
DRAFT, KEEL, AT ASSIGNED FBD	9.6 m (31'-7⅛")
GROSS TONNAGE, U.S.	13223
NET TONNAGE, U.S.	8008
GROSS TONNAGE, PANAMA	13653
NET TONNAGE, PANAMA	9966



TYPICAL SUMMARY TABLES

DRY CARGO

HOLD	DECK	FRS.	GRAIN m^3	BALE m^3	VCG m	LCG m
1	MN.	17-37	425	395	17.3	62.0 F
2	2' ND	36-54	807	741	13.3	48.6 F
TOTAL, 7 HOLDS			16315	15062	9.6	2.3 A

CONTAINERS

HOLD	DECK	FRS.	TIERS	NO.	CAP. m^3	VCG m	LCG m
4	MN.	76-103	1	18	570	17.4	14.9 F
4	TK.TOP	76-103	6	108	3419	8.8	14.9 F
TOTAL				126	3989	10.1	14.9 F

REFRIGERATED CARGO

HOLD	DECK	FRS.	NET CAP. m^3	VCG m	LCG m
5	2' ND	137-147	196	12.3	25.1 A
5	26,5' FLT.	134-147	267	9.4	23.6 A
TOTAL			732	9.1	24.0 A

PERMISSIBLE DECK LOADINGS

TYPICAL

DECK	FRAMES	TONNES/m^2
TANK TOP	14-36	3.72
TANK TOP	75-106	7.00
MAIN DK.	14-208	2.50

CARGO OIL

COMPT.	FRS.	TONNES	VCG m	LCG m
D.T. #1, S	14-28	94.3	5.0	65.3 F
D.T. #1, P	14.28	94.0	5.1	65.3 F
TOTAL		1489.0	3.7	3.3 F

FUEL OIL AND BALLAST

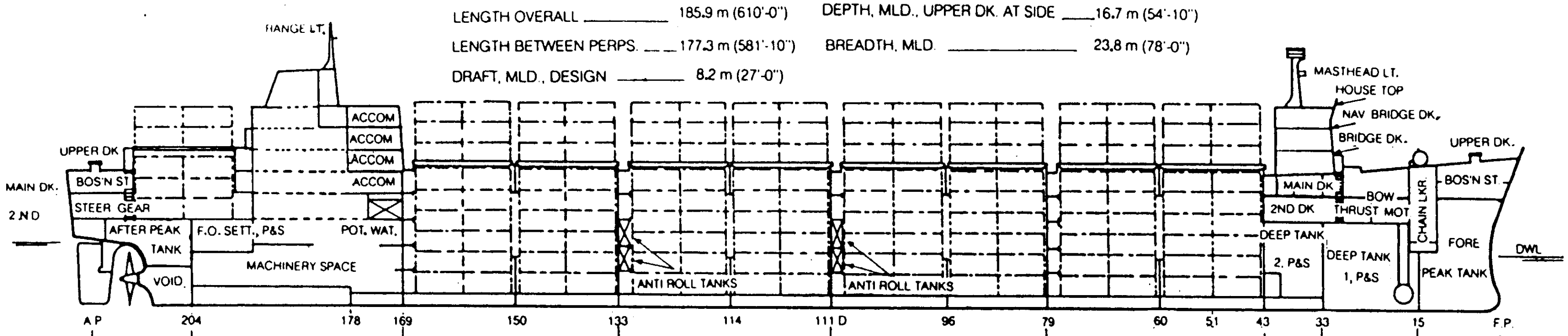
COMPT.	FRS.	FUEL TONNES	BALLAST TONNES	VCG m	LCG m
FOREPEAK	0-14		112.5	3.6	61.8 F
1A DB	14-24	49.4	53.7	1.4	67.1 F
TOTAL		4863.0	5072 0	2.5	4.3 A

NOTE: L.C.G. FOR'D (F) AND AFT (A) MEASURED FROM AMIDSHIPS, 81.7m (268 FT) FOR'D OF A.P.

Fig. 39 Capacity plan—multi-purpose dry cargo ship

PRINCIPAL PARTICULARS

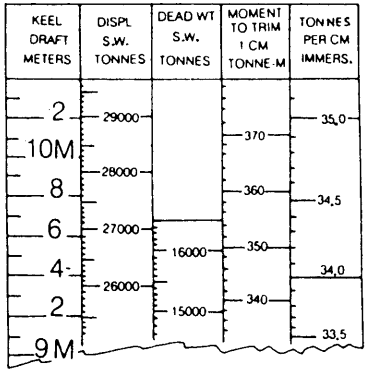
LENGTH OVERALL ____ 185.9 m (610'-0")
LENGTH BETWEEN PERPS. ____ 177.3 m (581'-10")
DRAFT, MLD., DESIGN ____ 8.2 m (27'-0")
DEPTH, MLD., UPPER DK. AT SIDE ____ 16.7 m (54'-10")
BREADTH, MLD. ____ 23.8 m (78'-0")

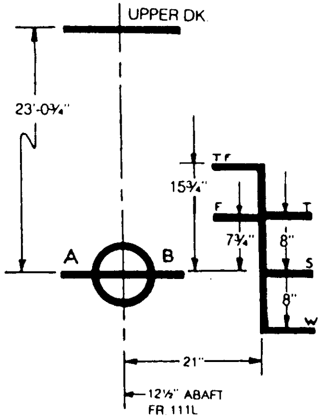


CHANGE OF DRAFT AT DRAFT MARKS FOR'D AND AFT IN CM FOR 100 TONNES ADDED WEIGHT AT SPECIFIC LOCATIONS THROUGHOUT SHIP LENGTH

FOR'D	-9.5	-8.5	-7.5	-6.5	-5.6	-4.4	-3.7	-2.7	-1.7	-0.7	-0.2	+1.2	9.1 m (30 FT) MEAN DRAFT	+11.2	+12.2	+13.2	+14.2	+15.2	+16.1	+17.0	+18.0
AFT	+11.6	+10.9	+10.2	+9.6	+8.9	+8.3	+7.6	+6.9	+6.2	+5.6	+4.9	+4.4		-2.9	-3.6	-4.2	-4.9	-5.5	-6.1	-6.7	-7.4

HOLD NO. 4 BALLAST TANK — HOLD NO. 3 BALLAST TANK — HOLD NO. 2 BALLAST TANK — HOLD NO. 1 BALLAST TANK





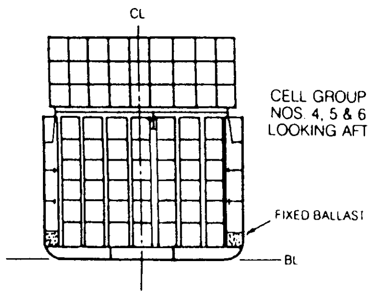
TYPICAL SUMMARY TABLES

SUMMARY OF CONTAINERS

SPACE	FRAMES	CG ABV.BL m	CG ABAFT FP m	NO. OF CONTAINERS
CELL GROUP NO. 1 TIERS 1-2-3	43-60	6.1	35.4	14
CELL GROUP NO. 1 TIERS 4-5-6	43-60	13.5	35.4	26
CELL GROUP NO. 1 TIER 7-9 ABV. DK	43-60	21.8	35.4	48
CELL GROUP NO. 9 TIER 1-2-3	197-216	14.6	165.3	24
CELL GROUP NO. 9 TIER 4-5 ABV. DK.	197-216	22.4	165.3	32
TOTAL		12.1	90.3	1070

SHIPS STORES

SPACE		FRAMES	CG ABV.BL m	CG ABAFT FP m	NET CAPAC. m^3
BOSUN'S STORES	MAIN DK.	STEM-11	15.9	3.1	91.6
BOSUN'S STORES	MAIN DK.	220-STERN	14.9	163.9	57.1
STOREROOM (STBD)	MAIN DK.	201-204	15.2		
TOTALS			14.1	129.3	391.2



CELL GROUP NOS. 4, 5 & 6 LOOKING AFT

NOTE:
MAX. WEIGHT OF EACH STACK OF 6.1 m (20-FT.) CONTAINERS ON HATCH COVERS LIMITED TO 24.5 TONNES (54,000 POUNDS).

FIXED BALLAST

HOLD	CONCRETE m^3	CONCRETE TONNES	STEEL TONNES	TOTAL TONNES	CG ABV. BL m	CG ABAFT FP m
1	484.5	1132.7	3.9	1136.6	2.8	42.5
2	309.8	724.4	2.9	727.3	2.8	67.7
3	204.6	478.4	2.6	481.0	2.8	96.3
4	460.3	1076.0	2.3	1078.3	2.8	123.2
TOTAL	1459.2	3411.5	11.7	3423.2	2.8	80.8

Fig. 40 Capacity plan—containership

(Continued from page 51)

markings and the Plimsoll mark are shown at the top of the draft scale. (The assignment of freeboard is covered in *Ship Design and Construction,* Taggart, 1980). A scale of deadweight is also shown. A trim table is often included to permit the estimation of changes in trim resulting from the addition or removal of weights at various locations along the ship.

Included on the capacity plan may be a list of permissible deck loadings, in tons per m^2, and the outreach and lifting capacity of cargo handling cranes or booms with which the ship is fitted.

The ship shown in Fig. 39 carries general cargo in five holds, containers in guides in one hold, refrigerated cargo in one hold, and cargo oil in deep tanks under four holds and outboard of one hold. Deck-stowed containers, loaded by a shipboard gantry, are regularly carried on the hatch covers over one hold.

The capacity plan for a typical all-container ship is shown in Fig. 40.

8.3 Cargo Capacities. Detail calculations are undertaken to determine the volumes of individual spaces. In the preliminary design phase, approximations to capacities may be adequate. When the design is finalized, exact methods should be used. Where spaces are composed of simple geometrical forms, the standard geometrical formulae may be used. However, on most ships there are numerous spaces bounded on at least one side by the curved hull surface, which are more amenable to calculation by one of the rules of integration.

Two types of capacity are customarily listed on the capacity plan for general cargo holds:

(a) Bale capacity represents the volume below deck beams and inboard of cargo battens which is available for stowing the typical commodities found in general cargo, usually in the form of bales, barrels, bags, crates and boxes.

(b) Grain capacity represents the net molded volume of the space, after deductions for the volume of structure and of ceiling, that is available for carrying granular cargoes in bulk.

In order to calculate bale capacity, a number of frames are selected between the bounding bulkheads. Sectional views are then drawn which show the inside of cargo battens, bottom of deck beams above, and hold ceiling at each frame, as shown in Fig. 41. The cross sectional area available for stowage in way of each frame is then found; these allow a sectional area curve to be drawn, which may be integrated using Simpson's Rule to find the volume, and transverse and longitudinal centroids. Horizontal sections through the space may be taken at a series of levels, and integrated vertically to find the vertical moment, and centroid for partially full and 100 percent full condition.

For grain capacities, the transverse areas are taken to the molded lines, except that deduction is made for ceiling on the inner bottom. A deduction for shifting

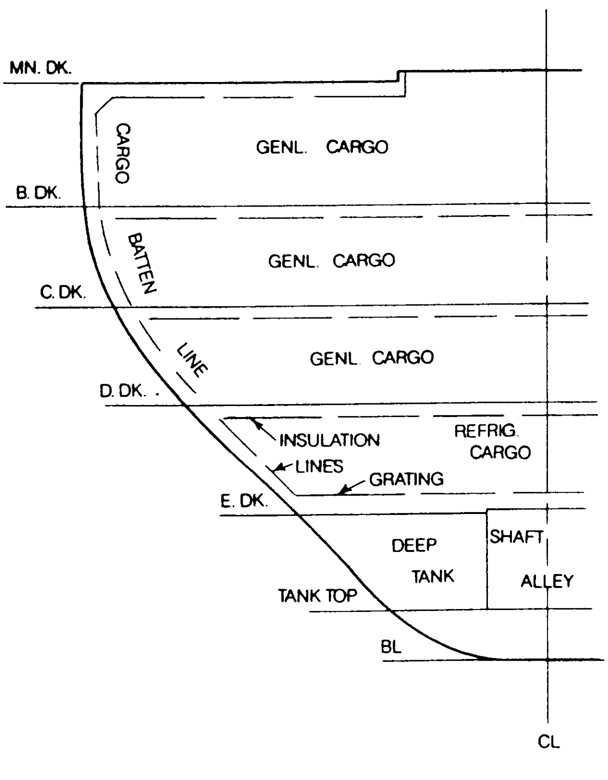


Fig. 41 Section for capacity calculaitons

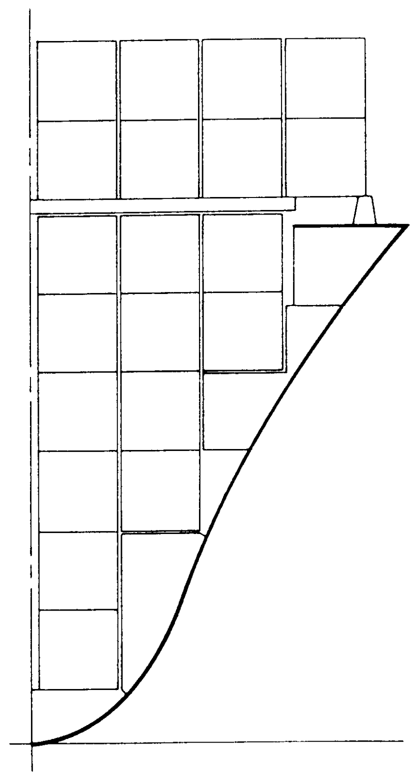


Fig. 42 Section showing container clearances

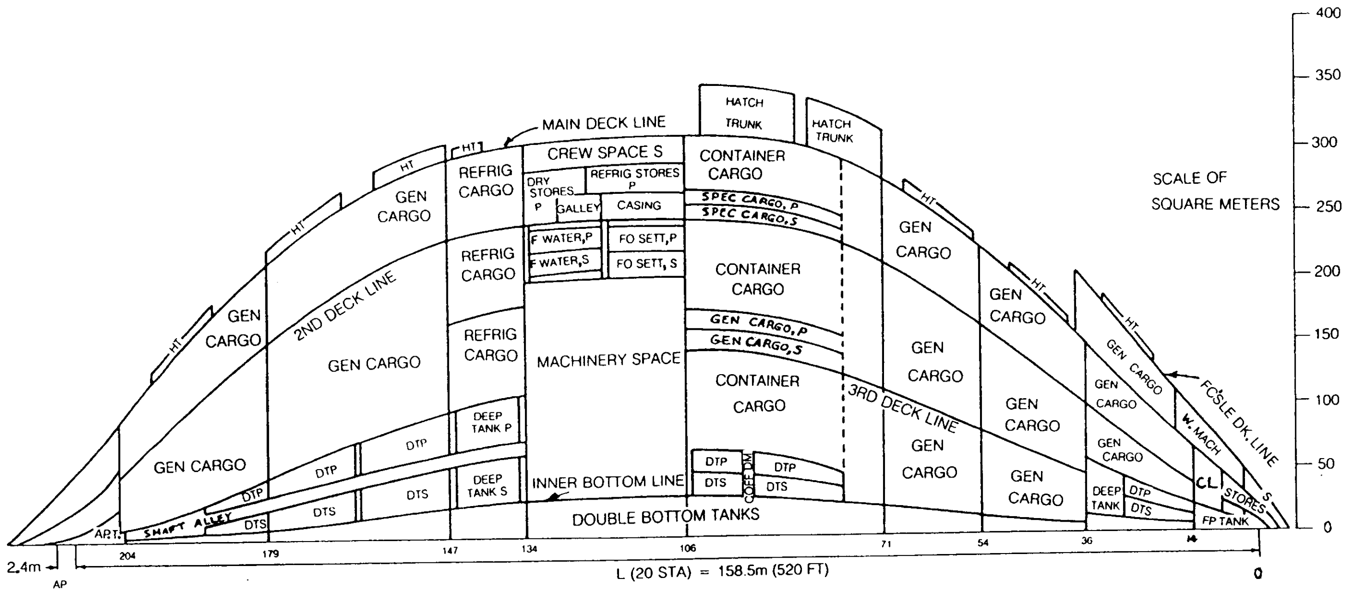


Fig. 43 Underdeck area curve—dry cargo ship

boards may also be necessary. For both bale and grain capacities, deductions must be made for stanchions, pipe covering, deck gratings and other such interferences with stowage.

Refrigerated cargo compartments are figured for total capacity inside of insulation, with deductions for grating on decks, for stanchions, batten protection of refrigerating coils, for air ducts and fan rooms. This volume is usually from 60 to 80 percent of the molded volume. When refrigeration is for fruit carrying, and bins are provided for stowing the fruit, a separate bin capacity is figured, this being the net inside capacity of the bins. The capacity plan, Fig. 39, shows refrigerated cargo forward of hold No. 6. The space taken up by insulation is evident.

Thus far it has been assumed that all cargo is homogeneous; that is, that each cubic meter of it weighs the same. This is by no means always the case. In figuring the centers of gravity of weights in mail rooms, baggage rooms and special storerooms, it may be desirable to estimate the centers of gravity of the weights as they are actually expected to be placed. For some services, it may be desirable to figure that cargo of different weights per cubic meter may be carried in different parts of the vessel. If cargo is hung from the overhead, as for example meat in refrigerated compartments, the center of gravity is effectively at the hook.

When the cargo consists of containers, barges or vehicles, the problem is different, inasmuch as each unit occupies a finite and predictable space. Cross sectional drawings through the container or barge holds, or vehicle decks, showing how the units are to be stowed, are more useful than volumetric calculations. A capacity plan for such vessels shows the number, size and sometimes limiting weight of units, and where they are to be stowed on the ship, rather than the volume of the holds. It is important in the design phase to demonstrate that the lower outboard corner of such specific units and their supporting structure, can be fitted in the space available inside the molded line of the hull, as shown by Fig. 42, and that the number of tiers of units to be stowed in the hold can be accommodated under the hatch covers. In the case of roll-on/roll-off vessels, outline drawings of the vehicles as stowed are sometimes shown to demonstrate available clearance between pairs of lanes and between the tops of vehicles and overhead structure.

A useful drawing sometimes prepared in studies of capacity is a curve of underdeck areas. This is quite similar to a sectional area curve, such as shown in Fig. 24, and has the same ordinate and abscissa units. However, the curve plots sectional areas below the main deck. It may be constructed by the use of Bonjean Curves, modified as necessary for volumes in way of trunks or hatches. The space below the curve is subdivided by lines representing decks and bulkheads, so that all internal spaces are accounted for. Fig. 43 shows an example of such a curve for the dry cargo ship shown in Fig. 39 simplified to illustrate the principles. Many spaces across the ship are made up of readily calculable rectangles, and the cross section of the wing tanks with curved boundaries can sometimes be found by subtracting the rectangular areas from areas bounded by the curve ordinates. An underdeck area curve may be used to check available space in the early stages of design, not only from the point of view of capacity and payload, but also to check tonnage volumes. Tonnage is covered in *Ship Design and Construction* (Taggart, 1980). In the case of certain recent

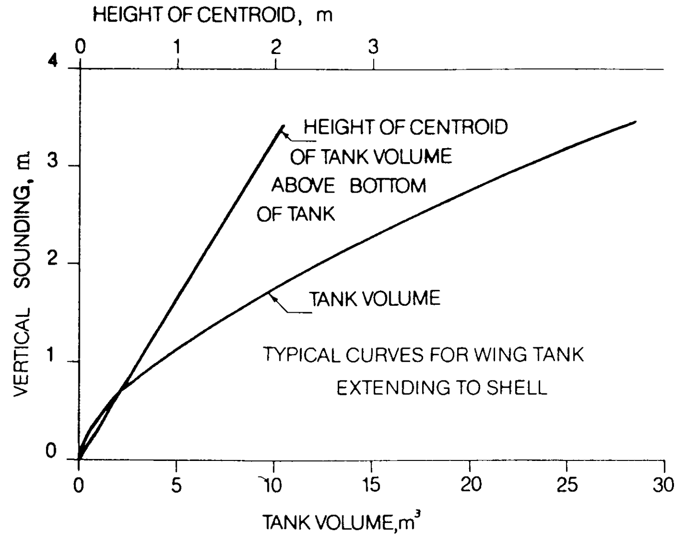


Fig. 44 Tank capacity curve

naval ship designs, internal volumes are at a premium to meet requirements for habitability and electronics. An underdeck area curve is a useful means of determining whether such requirements can be met.

8.4 Tank Capacities. The volumes of rectangular or cylindrical tanks, and their centroids, for any percentage of filling, are readily determined using standard formulas. In the case of irregularly shaped tanks like wing tanks in parts of the ship where the vessel's shape is changing rapidly, the volumes may be found using the method suggested in the preceding section, with the exception that the cross sectional area should be taken from longitudinal boundaries outboard right to the molded surface, rather than to the cargo batten line, as well as between the molded lines of the decks and flats which form the upper and lower boundaries of the tank. Deductions for the structure and other items are then made according to the data in Section 8.5.

Tank capacity tables generally give the tank volumes at a series of closely spaced depths, allowing a curve of volume vs. depth of sounding (depth below surface) to be drawn, as in Fig. 44. The calculations are repetitive and so are adaptable to computer programming. The basic inputs needed are the ordinates from the inboard boundary out to the molded shell line at a series of elevations and frames.

8.5 Deductions from Tank Volumes; Allowance for Expansion. Only molded tank volumes have been discussed thus far. In all tanks there are various internals, such as the frames of the vessel projecting into the tanks, longitudinals and floors in double bottoms, stiffeners on bulkheads and swash plates. There will also be various local deductions and additions which must be calculated separately. For the miscellaneous structural internals, a percent deduction is usually made. Typical data are given in Table 14.

The table gives also the usual allowance for the expansion of petroleum products. The practical operating capacity of an oil tank is not its total net capacity, but from 2 to 5 percent less, since, if the tank could be completely filled with cold oil, the oil would expand and overflow the tank when it becomes warm. The tank capacity is, therefore, calculated as being 95 to 98 percent full, depending upon the usual practice of the owner. Common standards are 95 percent for U.S. Navy practice, and 98 percent for U.S. merchant ma-

Table 14—Typical Corrections to Calculated Tank Capacities

Type of Tank	Allowance For Expansion	Directly Calculated Deductions	Directly Calculated Additions	Average Percent Deductions
Fuel Oil	2 to 3 percent U.S. Navy uses 5 percent	Pockets, sea chests, bulkhead corrugations, heating coils, structure in tank	High coaming hatches, expansion trunks	Double bottom tank without heating coils, 2-1/4 to 2-1/2 percent of molded capacity; add 1/4 percent if with heating coils
Fresh water; salt water ballast	None	Trunks, pockets, sea chests, bulkhead corrugations, structure in tank	High coaming hatches	Double bottom tank, 2-1/4 to 2-1/2 percent of molded capacity
Cargo oil	2 percent	Trunks, pockets, bulkhead corrugations, cargo oil piping, heating coils, structure in tank*	High coaming hatches, expansion trunks	1 percent of molded capacity

* Special considerations may require separate calculations for various levels in tank.

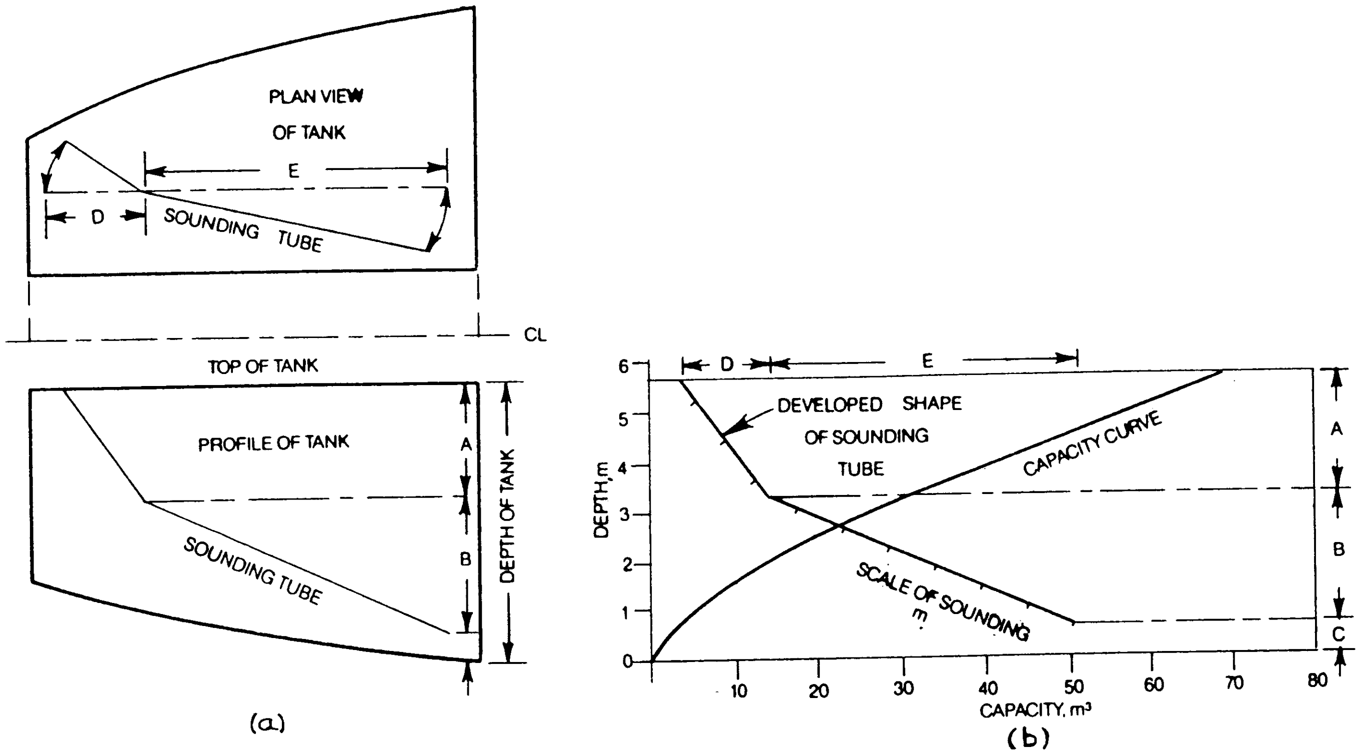


Fig. 45 Sounding tube geometry

rine practice. But more than one percentage is sometimes used on capacity plans. The actual values should, of course, always be stated.

8.6 Capacity Curve and Centroid. The vertical height to the centroid of the contents of any space above the bottom of the space may be readily found provided there is available a capacity curve extending vertically from the bottom of the space. This allows use of the method for VCB of Sec. 5.9.

In Fig. 44 the height of the centroid above the baseline of the curve is obtained by taking the area to any level between the capacity curve and its horizontal base line, divided by the area of the circumscribing rectangle to that level, and multiplying the result by the height from the base line of the curve to that level. The areas may be found by planimeter, or by numerical integration.

8.7 Soundings and Sounding Tables. When the amount of liquid in a tank is determined by lowering a rod or weighted tape measure through a sounding tube, or by any device which otherwise senses the level of the liquid free surface along a line extending into the tank, the sounding is interpreted by use of a table or curve of net tank volume vs. depth, as noted in Section 8.4 and shown in Fig. 44.

It frequently happens that only certain locations are feasible for the upper end of a sounding tube and that a straight vertical pipe from that location will not reach

Table 15—Weights and Conversion Factors

Quantity	Water		Oil				
	Salt	Fresh	Fuel	Diesel	Lube	Gasoline	DFM*
m^3 per t (a)	0.975	1.000	1.059	1.156	1.198	1.393	1.199 − 1.178
Barrels per t (b)	6.139	6.296	6.663	7.277	7.541	8.768	7.548 − 7.417
Gallons per t (c)	261.8	268.5	279.6	305.4	316.4	367.9	316.8 − 311.2
Cubic feet per Long Ton (d)	35	35.9	38	41.5	43	50	43 − 42.3
Pounds per cubic foot	64	62.4	58.95	53.98	52.09	44.80	52.04 − 59.97
Barrels per Long Ton	6.24	6.40	6.768	7.391	7.658	8.905	7.673 − 7.538
Pounds per gallon	8.556	8.342	7.881	7.216	6.964	5.989	6.958 − 7.082

* Diesel Fuel, Marine or Distillate Fuel, Marine—U. S. Navy multi-use fuel.
(a) 1 tonne = 1000 kg; weight of 1 metric ton = 2204 pounds = 1t.
(b) 1 barrel = 5.61 cubic feet.
(c) 1 cubic foot = 7.48 gallons.
(d) 1 Long Ton = 2240 pounds.

Table 16—Stowage Factors, m^3 per metric ton

Item	Packing	m^3/t	Item	Packing	m^3/t
Apples	Boxes	2.23	Lead, Pig	Neat Stowage	0.22
Autos	Assembled and Uncrated	7.52	Lard	Boxes	1.25
Barbed Wire	Rolls	1.53	Machinery	Crated	1.39
Bauxite	Bulk	1.07	Meat	Cold Storage	2.65
Beans	Bags	1.67	Molasses	Bulk	0.74
Beer	Bottled in Cases	2.23	Newspaper	Bales	3.34
Butter	Cases	1.67	Nitrate	Bags	0.72
Canned Goods	Cases	1.34	Oil	Drums	1.25
Carpets	Bales	3.90	Oranges	Boxes	2.17
Cement	Bags	0.97	Oysters	Barrels	1.67
Cement	Bulk	0.72	Paint	Cans	1.00
Cheese	Crates	1.81	Palm Oil	Bulk	1.09
Citrus Fruits	Boxes	2.62	Paper	Rolls	2.51
Coal, Average	Bulk	1.32	Potatoes	Bags	1.67
Cocoanuts	Bulk	3.90	Poultry	Boxes	2.65
Coffee	Bags	1.62	Railroad Rails	Neat Stowage	0.42
Condensed Milk	Cases of Cans	1.23	Rice	Bags	1.62
Copper Ore	Bulk	0.47	Rope	Coils	2.51
Copra	Bags	2.09 to 2.37	Rubber	Bundles	3.90
Corn	Bulk	1.41	Rye	Bulk	1.62
Cotton	Bales, Average	1.45	Salt	Bulk	1.03
Currants	Crates	1.81	Silk	Bales	3.06
Dried Fruit	Boxes	1.25	Steel Bolts	Kegs	0.58
Dry-Goods	Boxes	2.79	Steel Sheets	Crated	0.42
Fish	Barrels, Iced	1.39	Sugar	Bags	1.31
Flour	Bags	1.34	Tar	Barrels	1.50
Furniture	Crated	4.35	Tea	Cases	2.79
Glass	Crated	3.62	Tile	Boxes	1.39
Gypsum	Bags	1.24	Timber	Oak	1.09
Hardware	Boxes	1.39	Timber	Fir	1.81
Hides	Bales, Compressed	2.23	Tung Oil	Bulk	1.07
Iron Ore	Bulk	0.30 to 0.53	Turpentine	Drums	1.59
Iron Ore Pellets	Bulk	0.25 to 0.53	Wheat	Bulk	1.31
Iron, Pig	Neat Stowage	0.28	Wheat	Bags	1.45
Jute	Bales	1.84	Whiskey	Cases	1.74
			Woodchips	Bulk	3.07

the lowest part of the tank. In such cases, it is usual to make the tube sloping and sometimes curved with a large radius. If the tube is sloping or curved, 1 m measured along the tube will not indicate a difference in level of 1 m vertically. In making a sounding table for such a tank, it is, therefore, necessary to allow for this difference. This is done as illustrated in Fig. 45(a). The sounding-tube installation is checked on the ship, as the drawing is usually only diagrammatic. The line of the tube as actually fitted is laid out on the same graph paper as the capacity curve. Any slope or curve in the line of the tube is developed into the plane of the graph paper so that the true shape and length of the tube are shown in the plane of the capacity curve, as illustrated in Fig. 45(b). Even by sloping or curving the sounding tube, it is not always possible to reach the very bottom of the tank. Consequently, the zero sounding as given in the sounding table often shows a considerable number of metric tons or barrels or liters in the tank.

In cases where it may be permissible for tanks to hold alternatively fuel oil or ballast, there should be capacity tables in metric tons of salt water and metric tons, barrels and liters of oil. Table 15 provides conversions for the various liquids and systems of measurement.

8.8 Effects of Heel and Trim. In the event a ship experiences heel or trim, the shift of free surface of the liquid in a tank in order to remain horizontal takes place in such a way that the location of the centroid of free surface area remains fixed with respect to the tank. Thus, a sounding rod experiences no change in reading due to heel and trim only if it passes through the centroid of the tank horizontal cross section. Inasmuch as physical constraints generally lead to a sounding tube location removed from the line of centroids, a trim and/or heel correction in tank sounding reading is generally called for.

It may be shown that the correction, in tank volume for trim is, in any consistent units,

$$A \cdot d \cdot \frac{t}{L}, \qquad (52)$$

where A is tank horizontal cross section area, d is longitudinal distance from centroid to sounding tube, t is trim and L is ship length between draft marks.

The correction is minus if ship trims by the stern (and sounding tube is abaft centroid), in which case the correction must be deducted from the tank volume.

The correction in volume for heel is,

$$A \cdot d \cdot \tan\phi, \qquad (53)$$

where d is distance of sounding tube to port or starboard of tank centroid and ϕ is angle of heel. The correction is minus if ship heels to starboard and the sounding tube is to starboard of the centroid.

In order to accomplish these corrections completely, the location both of the sounding tube and of the tank free surface centroid must be known at all elevations.

8.9 Ullage. The traditional way of determining the amount of liquid in the tank of a tank vessel is to lower a weighted chain with a scale on it until it touches the surface of the liquid and so measure the distance from the top of hatch to the free surface. This is called *ullage,* and the associated tables are called ullage tables. Capacities should be given in barrels or cubic meters, and weights in metric tons. Ullage tables differ from tables of tank capacity, in that tank capacities are given for varying depths of the liquid in the tank or soundings, while ullage tables give the tank capacity for differing amounts of ullage, or distance from top of tank to liquid surface.

Ships with inert gas systems to protect against explosion should have sealed and remotely operated means of reading liquid levels, inasmuch as the atmosphere within the tank is normally at a slight positive pressure.

8.10 Cargo Stowage Factors. The average specific volume or *stowage factor* of the cargo to be carried may exert a strong effect on the design of a ship. Low stowage factor cargoes, such as ores and finished steel, lead to ships which are weight limited, and tend to have large fullness to obtain a large displacement on fixed dimensions, and have low freeboard. High stowage factor cargoes, such as crated furniture, automobiles and containers tend to result in volume limited ships with high freeboard and relatively low fullness, in order to achieve adequate propeller immersion and draft on fixed dimensions. The volume requirements of containerized cargo are met on many containerships by carrying some of the containers on deck in several tiers, thereby providing the capability to carry enough cargo to reach design displacement when fully loaded.

Table 16 shows approximate stowage factors for a number of typical kinds of cargo. Thomas (1957) gives stowage factors for a wide variety of commodities.

8.11 Consumables. This term includes fuel oil, lubricating oil, fresh water for culinary and drinking purposes, fresh water for washing purposes, fresh water for boiler feed, and stores that are expended on the voyage, such as supplies and food provisions of all kinds. The capacity required for consumables depends upon the main propulsion power, the length of the voyage, the number of passengers and crew, and the quality of the accommodations provided. Owners usually have definite ideas as to the amount of provisions, stores and fresh water necessary for their service. When better information is not available, Table 17 gives reasonable assumptions for approximate weight per day per passenger and member of crew for stores, and liters of fresh water consumed per day per person.

Fresh water capacity will be dependent upon whether water is to be obtained ashore and carried for the length of the voyage, or the ship's distilling plant can meet all requirements at sea. In the latter case, requisite capacity for fresh water will be much reduced.

Many ships provide separate tankage for potable water, distilled water, and boiler feed water (if steam propelled). A survey of post World War II designs shows average fresh water tank capacities of 1.7 metric tons per person for potable water, and 0.0032 and 0.007 metric tons per shaft horsepower of propulsion machinery (maximum rating) for distilled water and boiler feed water, respectively.

Table 17—Weight Allowances for Stores and Water

Item	Kg per person per day	
	Passengers	Crew
Fresh water, moderate ships	40	20
Fresh water, luxury ships	100	45
Stores	10	5
Provisions	4.5	4.5

Baggage for a long voyage may amount to an average of 0.1 t per passenger and on luxury ships to as much as 0.17 t per passenger. For tourist class passengers, it may be as little as 0.07 t per passenger. For short voyages and excursions it will average about 0.08 t per passenger.

Table 18—Fuel Oil Consumption in Port, metric tons per day

Type of Propelling Machinery	Steam Turbine	Diesel
Source of Auxiliary Power	Steam	Diesel
Minimum Fuel Consumption	3.5	2
Add for each 100 t of cargo moved	0.8	0.5
Add for each 100 "tons" of refrigeration	1.2	0.9
Add for each 100 persons complement	0.6	0.4

For a steam tanker with steam-turbine driven cargo pumps about 0.5 t of fuel will be required for each 1000 t of cargo oil pumped. If cargo pumps are diesel-driven, about 0.3 t of fuel will be required. Direct calculations using specific fuel rate of pump prime mover are recommended for precise values.

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